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A STUDY OF OPERATING COSTS FOR "SLANT ANGLE" PALLETIZED WAREHOUSE STORAGE

A THESIS

Presented to

The Faculty of the Graduate Division

by

Otto Raymond Ellars, Jr.

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Industrial Engineering

Georgia Institute of Technology

December, 1967

ACKNOWLEDGMENTS

The writer wishes to thank Dr. Paul T. Eaton, his thesis advisor, whose guidance and assistance made the study possible, and also Professor James M. Apple and Dr. J. Gordon Davis, of his thesis committee, for their constructive criticisms and suggestions.

Special thanks go to Mr. E. W. Sammons for the use of the Atlanta Public Schools Warehouse in which the experiments were run, and to Charles Hemphill for his tireless driving.

Also, my appreciation to my mother, father, aunt, and Joan who all had to live through it.

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SUMMARY

"Slant angle"pallet placement had received very little attention prior to 1960. Since that time at least two quantitative studies have been made concerning space utilization using "slant angle" pallet placement. To the best of knowledge, no quantitative studies concerning operating efficiency had been made.

The objectives of this study were to investigate the possibility of operating cost savings through the use of "slant angle" pallet placement, to find the optimum "slant angle" if money savings were to be realized, to investigate the possibility of using wider aisles to lower operating costs more than the accompanying increase in space costs, and to combine the results of this operating efficiency study with the results of the space utilization studies that have been made.

The conclusion of the study was that significant operating cost savings were possible using "slant angle" placement, but that these savings were far less than those realized from a space efficiency study except in a small, high turnover rate warehouse. It was found not to be economically feasible to use wider aisles to lower total costs. To reduce operating costs to a minimum, the greatest "slant angle" that is permissible under other considerations, primarily space utilization, should be used because operating costs are linearly decreasing as the "slant angle" is increased. A combination of this study with a space utilization study showed that the space utilization study produced far greater money savings and that the optimum "slant angle" was not changed.

The figures used in making the study were obtained using a stop watch time study. Judging from the significant amount of money to be saved using "slant angle" pallet placement, it is recommended that it be used in any modern palletized warehouse.

CHAPTER I

INTRODUCTION

In our modern industrial society, particularly in the United States, warehousing is taking on an increased importance. As companies grow, producing more and more goods through improved production methods, better and more efficient warehouse design is needed. Incoming raw materials and outgoing products must be stored by the manufacturer. The distributor must store a large variety of finished products which are shipped to various retail outlets. To optimize the design of a warehouse, the costs incurred in the warehousing operation must be minimized.

The warehousing operation from a materials handling viewpoint includes the following activities: (1) unloading from carrier, (2) materials storage, (3) materials issue and distribution (pertinent only in a manufacturing plant), (4) finished goods warehousing (pertinent only in a manufacturing plant), (5) stock picking, and (6) loading operations onto an outgoing carrier. This list was drawn from a paper by James M. Apple for a conference/seminar[3]. The other primary function of the warehouse is the actual storage of the material.

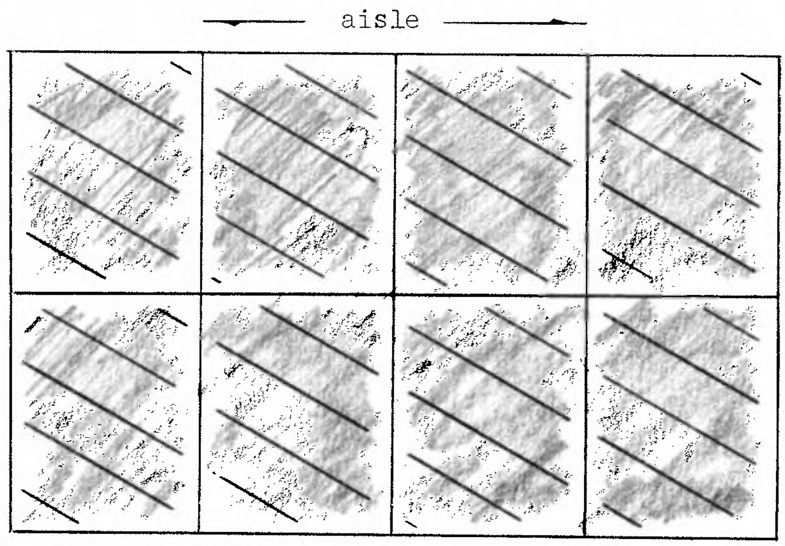
As time has progressed, warehousing has improved greatly. From the early days of manual handling and little or no layout consideration, the field has progressed to power fork trucks, conveyor lines, palletized unit loads, and reasonably well laid out warehouses. Still, there is much room for improvement.

Because warehouse operations do not increase the value of a company's product, they are not given as much attention as other areas within the company. Many warehouses are located in old buildings that were not designed to be warehouses and are poorly designed as such. This inhibits all of the functions of the warehouse, and adds to the problems of the warehouse designer.

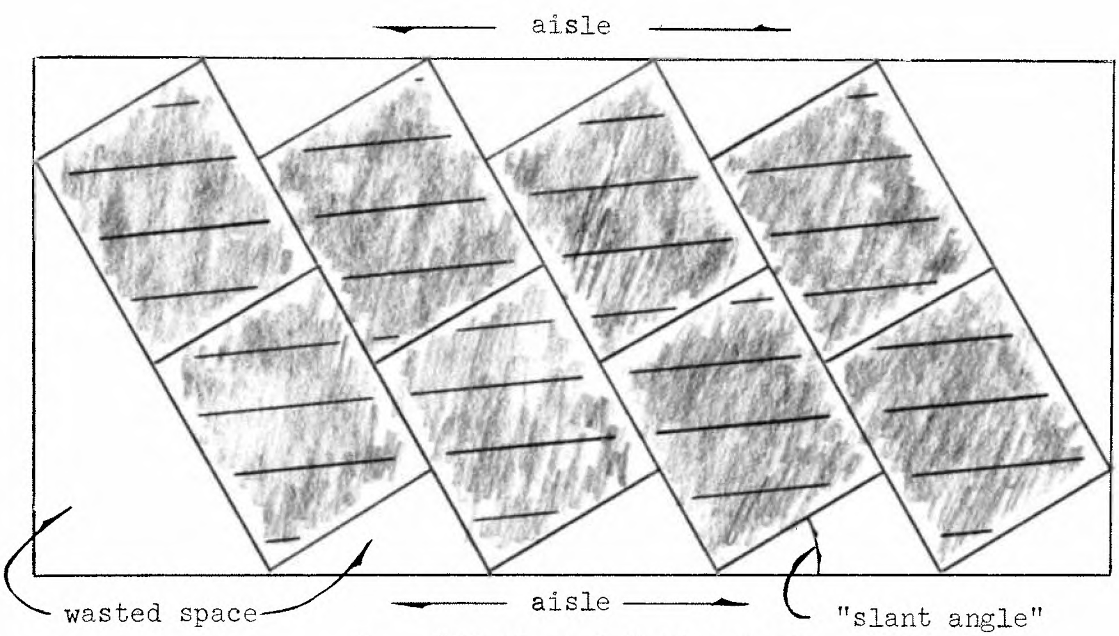
Two of the areas in which attempts have been made to reduce costs in warehouse operations are space economy and all moving operations involved in storing, picking, and removing the material from the warehouse. Space economy is concerned with actually placing as much material as possible in a given area. This involves the size and shape of the storage bays and the widths of the aisles. Attempts to reduce costs in the moving operations have primarily concentrated on developing more efficient and faster equipment.

As the art of warehousing advanced, another consideration came about, that of "slant angle" pallet placement. The ordinary method of placing pallets is to line them up along the aisle exactly perpendicular to the aisle, as in Figure 1-A. This method is referred to as stacking "on the square," "right angle stacking," "ninety-degree placement," or "zero-degree placement." In "slant angle" pallet placement each pallet is rotated a certain number of degrees with one corner remaining on the edge of the aisle, as in Figure 1-B. The "slant angle" is the number of degrees that the pallet has been rotated. As this angle is changed, space utilization and operating efficiency vary considerably.

"Slant angle" pallet placement was ignored mainly because it did not seem to shorten handling times enough to warrant what appeared to be a



A. Right angle placement



B. "Slant angle" placement

Figure 1. Right Angle and "Slant Angle" Pallet Placement

considerable loss in space efficiency. What caused this "appeared" loss was the large number of wasted space areas (including the space at either end of the storage bay) generated by the "slant angle" layout. (See Figure 1-B.) However, no quantitative studies had been made to prove or to disprove these ideas, so interest in "slant angle" pallet placement was dormant for some time.

In the last five years, to the best of knowledge, two studies have been made on the floor space utilization of "slant angle" pallet placement with directly conflicting results. These two studies attempted to determine an optimal angle at which to place pallets to maximize floor space utilization. However, due to different initial assumptions, one[19] came up with the result that right angle placement was the best while the other[4] arrived at a figure of around 35 degrees as being the optimum angle at which to place the pallets.

No quantitative studies have been made concerning a comparison between operating costs for right angle placement and "slant angle" placement. Since moving the stored material is one of the most important functions of a warehouse, it seems that the operating costs for moving this material must be considered in conjunction with the space utilization studies.

If an optimal angle for placing and removing pallets can be found and a significant time difference between right angle and "slant angle" placement exists, a dollar savings in operating costs can be realized. In addition to finding this optimal angle for operating efficiency, an optimal aisle width should be found, balancing the savings possible in operating costs using a wider aisle with the additional floor space costs incurred with the additional aisle space.

After the optimum angles have been found for space utilization and operating efficiency and the optimal aisle width has been found, each should be weighed according to the money savings possible. Then an optimum solution for the particular situation could be found. Since no conclusion has been arrived at concerning the best angle for space utilization, some studies were necessary to establish this.

There are many other factors evident when considering "slant angle" placement versus right angle placement. These should also be brought to light and weighed as to importance.

A factor which needs to be considered for both types of storage is that of clearance between the stored pallets. One of the studies[19] on space efficiency of "slant angle" placement took this into consideration and arrived at the conclusion that zero clearance was the optimal solution for maximum space utilization. Some clearance is necessary to move the pallets, however, and more clearance is necessary to allow for error in placing the pallets and for disorderly pallet loading. Also, various containers, such as carton boxes into which most products are packed tend to weaken and partially collapse, thereby necessitating additional clearance space. This clearance space will vary with every warehouse, depending upon the products handled, the climate, the pallet loading, and the training and experience of the fork truck drivers. It is doubtful that a single optimum clearance could be found that would be applicable to every situation.

Objectives

The primary objectives of this study are as follows:

1. To test the hypothesis that "slant angle" pallet placement will yield significantly lower handling times for moving material in the warehouse (in essence, the actual storing and picking operation) as opposed to right angle pallet placement.

2. To show if the handling times are lower, that a significant dollar savings in operating costs may be realized by using "slant angle" placement.

3. To find the optimum "slant angle" for minimizing operating costs.

4. To study aisle width in conjunction with the "slant angle" to see if larger aisle widths will lower operating times and costs enough to offset the increase in floor space cost (caused by poorer space utilization).

5. To combine the results of the operating cost studies (an optimum "slant angle" for operating cost reduction and the optimum aisle width) with the results of previous studies concerning space utilization.

6. To find a practical solution minimizing costs in the warehouse while giving consideration to each of the aforementioned areas.

Assumptions

In making this study it was necessary to make some assumptions concerning facilities and layout. These assumptions are applicable to most modern warehousing facilities. They are as follows:

1. The material handling equipment used is a standard four-wheel

rear steering rider type fork truck of 3,000-pound capacity.

2. All storage is on pallets of the same dimensions and placement is uniform and consistent within each bay.

3. All pallets are uniformly loaded and placed adjacent to one another with no interference (allowing a fixed clearance between pallets). They may be stacked.

4. The material to be moved has a turnover rate such that material handling costs are an appreciable cost factor.

5. Aisle width is not determined arbitrarily, but is calculated from pallet size, truck size, (turning radius,) and the "slant angle" at which the pallets are placed.

6. Neither the loading capacity of the truck nor the mobile functions of the truck are affected by varying the "slant angle."

7. Bay size is determined by needs and no allowance is made for loss of floor space due to columns and other obstructions.

8. The use of a "slant angle" does not affect the variation in times for different depths into bays or different stacking heights.

9. The warehouse building is of a suitable design and size so as not to interfere with the general layout with the exception of necessary columns.

Literature Search

For the reader who is not familiar with warehousing in general and with the use of fork trucks in warehousing, many articles and sections of books have been written on this subject. For introductory purposes, sections in books by Apple[3] and Briggs[6] will be informative. For more advanced reading on the subject of material handling and its impact on the productivity of a company, Eaton, Yale, and Towne, Inc. has published three short booklets[8,9,10] with articles written by leading men in this field today. These articles show how, through proper application of sound material handling policies the productivity and profits of a company may be increased. In the area of fork trucks, a booklet has been published by the Industrial Truck Association[14] showing most types of equipment available, applications of the equipment, and ideas on the selection of equipment. A program for training fork truck drivers has been published by the International Material Management Society[15] giving a complete training program including safety considerations. Many other interesting and informative books and articles have been written on the subject. The government[20] has published a number of articles on material handling and warehousing, also.

A thorough search of the literature on palletized warehousing has turned up very few quantitative articles on the subject of "slant angle" storage with no articles found concerning a quantitative study of the operating costs involved. Also, letters inquiring about any such studies were sent to many of the leading men in this field throughout the country with no affirmative replies. Letters were sent to leading fork truck manufacturers and U. S. Government sources with no affirmative replies. Following are descriptions of several articles that lead up to the present day status of "slant angle" pallet placement.

An article written in 1952 by E. E. McVeigh[16] was the first to make use of quantitative methods in the "slant angle" field. He points out the benefits of oblique, as he calls it, stacking: "(1) narrower aisles with less waste space; (2) faster, easier stock selection; (3) easier inventory checks." He points out that because of the smaller aisle requirements either the aisles may be made narrower or larger trucks may be used, thereby increasing the amount of material carried per load. He also points out that the operator can spot the merchandise much faster and then carry out the actual pickup faster using "slant angle" placement. He points out four factors warehouse managers (or designers) should consider when using "angle stacking." They are:"(1) aisle layout, (2) column locations, (3) pallet size, and (4) capacity of trucks." He claims that oblique stacking increased the number of pallets stored by 16 percent and speeded up operations by 25 percent, but there are no data or calculations to support these statements. He does present two graphs. The first shows the relationship between aisle width and amount of slant. The graph shows values of slant running from zero to about 37 degrees with the corresponding aisle width requirements plotted. He does this for 1,000-pound through 6,000-pound capacity fork trucks, and the resulting graph shows a linear relationship for all cases. The saving in aisle space averages 61 inches for the 37 degree case versus the right angle case. In the other graph he shows the relationship between the amount of slant and the area required by one pallet plus one-half of the adjoining aisle space for six truck sizes. This graph shows a space saving for "slant angle" placement. This is the extent to which he carried his quantitative studies with very few explanations and no quantitative work done on times or labor costs.

The next paper concerning "slant angle" placement was a master's thesis in 1961, written by H. M. Thornton[19]. It is primarily concerned with space efficiency, but does briefly mention the time and operating cost savings to be realized through slant angle storage. This thesis was later updated by Thornton and J. J. Moder[18] and will be discussed later.

In 1962 Dr. Donald J. Bowersox[5] published his views on the subject. He presents a good description of the difference between the "slant angle" storage and right angle storage and then proceeds to point out all of the good and bad points for both methods of pallet placement. He then makes some very limited calculations on space efficiency and presents a few diagrams to illustrate his arguments. He presents very little data to back up what he says or to back up his assumptions. Dr. Bowersox later helps Mr. Ronald H. Ballou[4] with his master's thesis on the same subject.

Ronald H. Ballou[4], in his thesis, attacks the problem of space utilization (his argument is presented in two issues of Transportation and Distribution Management). The first article presents an overall picture of the problem and explains some of the advantages and disadvantages of "slant angle" layout. He also made a survey of the large warehouses in the country to try to find out what a "typical" warehouse used for aisle widths, bay length, bay depth, truck size, and what the most popular pallet size was. The results were that a "typical" warehouse would have bays that were ten pallets long by five pallets deep. It would have eleven-foot aisles including a safety clearance of 6-inches. Three thousand pound capacity fork trucks were the most popular and the 40 inches by 48 inches pallet size was the most popular. The survey also showed that 86 percent of the larger warehouses in the country use right angle placement. He explains the way his experiment was set up and the factors which he varied: "slant angle," bay length, bay depth, pallet size, truck size, and safety clearance. He then explains why space is saved using a "slant angle," the necessary assumptions for his study, and his conclusions.

In the second article Ballou illustrates the problem and explains how to set up a warehouse using "slant angle" storage. He then presents a detailed form to be used by the reader to compute the space efficiency for angles from zero to 60 degrees, thereby arriving at the best "slant angle" for the reader's own particular case. Finally, he presents a graph showing the results of his form when used on the "typical" case. The highest space efficiency for that case came at about 35 degrees.

Almost exactly a year later, J. J. Moder and H. M. Thornton[18] published their article on space utilization. This was a detailed follow-up to Thornton's master's thesis[19] in 1961, and also approached the problem almost entirely from a mathematical standpoint. Mathematical equations were derived for the space efficiency and both the clearance between the pallets and the "slant angle" were varied over a wide range. Graphs were also presented showing the effect of changing certain other variables. They, as did Ballou, varied the bay length, bay width, truck size, and pallet size. Their conclusions were different, however. They proved that the clearance between pallets should be zero (in practice, as close to zero as possible) and that the most efficient angle is zero degrees. The reasons for the difference in the results arrived at in this study and the results arrived at in Ballou's articles[4] lie in the initial assumptions. Mr. Ballou, in his study, considered an entire bay, surrounded by four aisles, when he varied the "slant angle." This allows a saving in space from all four surrounding aisles. Also, there is a great deal of wasted space at both ends of each day. Mr. Ballou calculated how many pallets could be fitted into this space and included these in his calculations. With these assumptions, maximum space efficiency is achieved at a "slant angle" of about 35 degrees. Variance from this figure, in this particular case, would be due to different bay sizes, different truck sizes, and different pallet sizes. Moder and Thornton[18], in their study, do not use the wasted space at the ends of the bays and consider only the one aisle into which the pallets are removed. Under these assumptions, maximum space efficiency is arrived at using right angle placement. It would seem that Mr. Ballou's assumptions would be more applicable to most of the bays in a warehouse, while Mr. Moder's and Mr. Thornton's assumptions would apply to bays placed along a wall or in a corner.

Perhaps the question of how the general overall layout of bays is arrived at has arisen. A master's thesis was written on this subject by J. B. Hemmi in 1963, in which eighteen different layouts were considered with regard to both space utilization and average time of delivery. He found that the design of the layout has an effect on operating times in both large and small warehouses, but that space utilization is only affected for small warehouses. Therefore, for large warehouses, only handling times should be considered in selecting the most efficient layout. He then further points out which layouts have the lowest handling times. This study provides a start in selecting the proper overall layout for maximum warehouse efficiency.

An article by D. Frazier[11] in 1961 discusses the case of placing pallets with the greater dimension as the depth versus the case of placing pallets with the greater dimension being the width. It is concluded that greater depth is more desirable. The only drawback is that it is more difficult for order pickers to reach articles to the rear of the pallets. Thornton[19], in his master's thesis, and Moder's and Thornton's[18] article show that this is true in their mathematical derivation of space utilization formulae.

The literature search turned up many interesting and informative articles, but no previous studies concerning operating efficiency or operating costs as related to "slant angle" placement in the pallet size warehouse field.

CHAPTER II

PROCEDURE

The main objective of this study is to find what effect varying the "slant angle" of stored pallets has on the time necessary to place and remove the pallets. To discover this effect it is necessary to compare the times for various "slant angles" with the time for right angle placement. The first angle studied is zero degrees, as this provides the basis for the comparisons to be made. Then the "slant angles" were selected.

The "slant angles" studied started with twenty degrees, chosen for several reasons: (1) Any angle less than twenty degrees will not allow a significant reduction in the aisle width. This would mean that there would be very little, if any, space savings. (2) Any angle under twenty degrees would be very similar to a right angle placement from the driver's viewpoint. He would not be able to see the empty space any more clearly than he would with right angle placement and would have almost the same problem lining up on the opening, thereby taking almost as much time as with right angle placement. The largest angle was dictated by the space utilization factor. In his thesis, Thornton[19] pointed out that after the pallet slant angle reaches a certain degree a pallet will "mask" the pallet next to it. This angle is in the range of fifty degrees, depending on the pallet dimensions. Also, as is pointed out by Thornton[19] and Frazier[11] (for right angle placement), aisle frontage space is an important consideration. As the "slant angle" is increased, a longer and longer aisle is needed to serve a given number of pallet slots.

Ballou's article[4] also showed that 45 degrees is well above the "typical" optimum angle for optimum space utilization. For these reasons, 45 degrees was chosen as the upper limit. The study covered, therefore, "slant angles" of 20 degrees, 30 degrees, 35 degrees, 40 degrees, and 45 degrees.

During preliminary studies it was noticed that the aisle width restricted the driver's speed when he was entering and leaving a pallet slot. Since this is the area in which the time saving is to be realized it was decided to include in the objectives of the study a consideration of various aisle widths for each angle to be studied. This brought to light the problem of determining the proper aisle width for each "slant angle." As seen in Figure 2, when the angle is zero degrees or relatively small, the critical area is at points A and B. Before the driver can start to turn his wheels into the aisle, point A must reach point B. If the "slant angle" is large, however, a different problem is encountered. While the turning radius of the outside of the "rear" of the fork truck is fairly small (63 inches), the turning radius of the inside "front" tire is about 4 inches (estimated from Appendix A and experience). This means that the point (P in Figure 3) about which the truck turns is only 4 inches from the side of the truck. (It will vary, but will never be much larger except on very large trucks.) Consequently, as we can see in Figure 3, points A and B are no longer critical. The driver must back up until the pivot point (P) is very close (less than 4 inches in the test case) to the corner of the next pallet before he makes his turn. The critical area is now next to

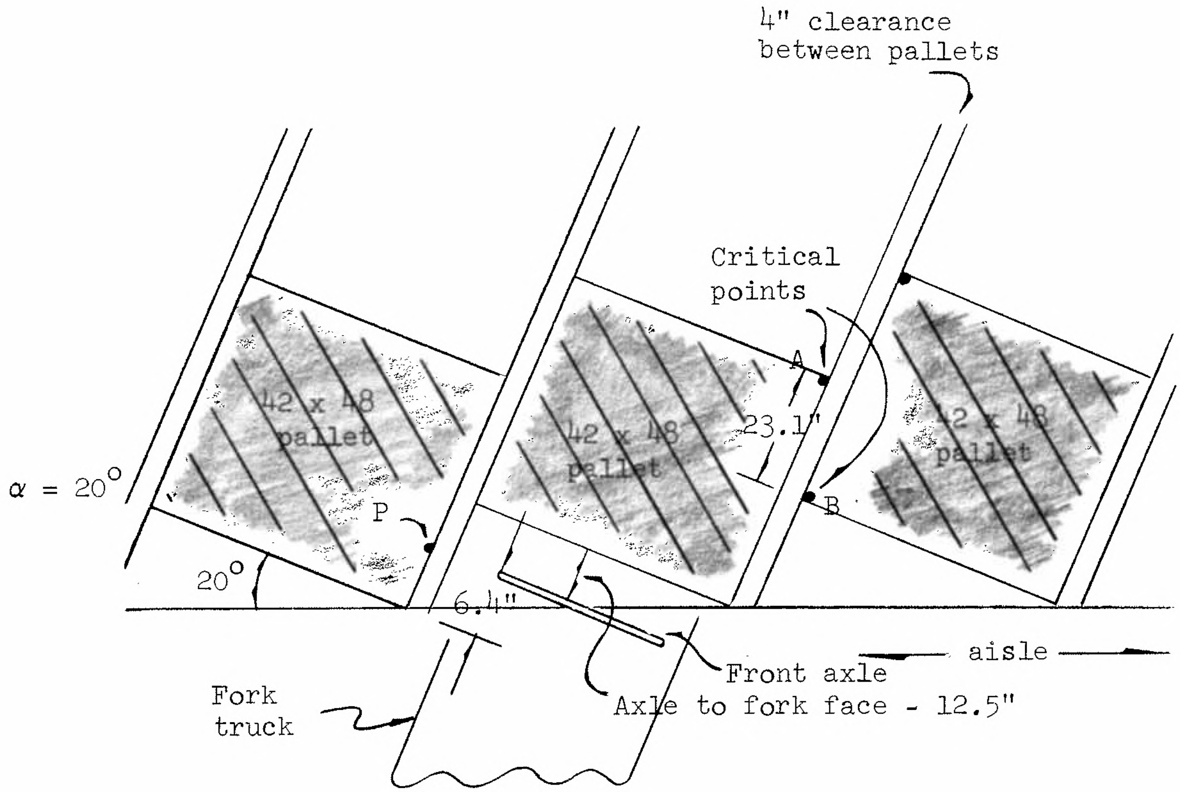


Figure 2. Pallet Removal for Small "Slant Angle"

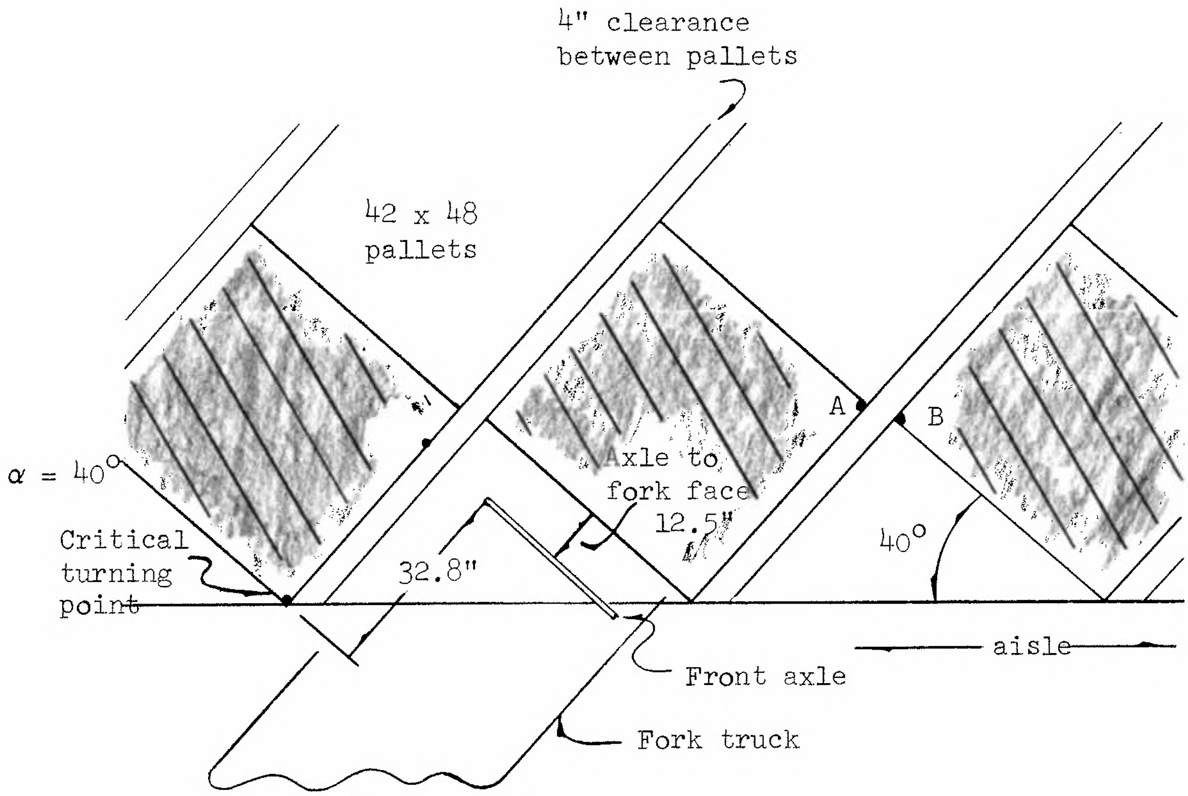


Figure 3. Pallet Removal for Large "Slant Angle"

the pivot point. There is also a critical angle at which the two previously mentioned cases meet, as illustrated in Figure 4. Through simple plain geometry manipulations this angle may be found and a general equation as follows may be derived (see Appendix B).

$$\alpha_c = \tan^{-1} \frac{(\text{pallet depth + axle to fork face})}{2(\text{pallet width + clearance between pallets})} \qquad (1)$$

This equation will hold as long as the pallet width is greater than the width of the fork truck. When calculating the aisle width one should not consider the clearance space between adjacent pallets. The driver of a fork truck will use this space to make the turns more easily, but its only reason for being there is to allow for error on the driver's part and to allow for disorderly pallet loading.

Thornton[19] derived in complex mathematical terms a similar formula for α_c using the width of the fork truck, the turning radius of the rear of the fork truck, and the turning radius of the "front" wheels. Because the warehousing operations are not done with the utmost precision it would seem that a geometric interpretation would be very close to the actual value and much more easily explained. It would also seem that the angle α_c would be a function of the size pallet being used, as well as the fork truck dimensions. From the same type geometric manipulations a formula was determined (Appendix B) to compute the required aisle width for a given "slant angle," fork truck, and pallet size. This formula is only valid for $\alpha < \alpha_c$ (where α is the "slant angle" and α_c is the angle in Figure 4.) It is as follows:

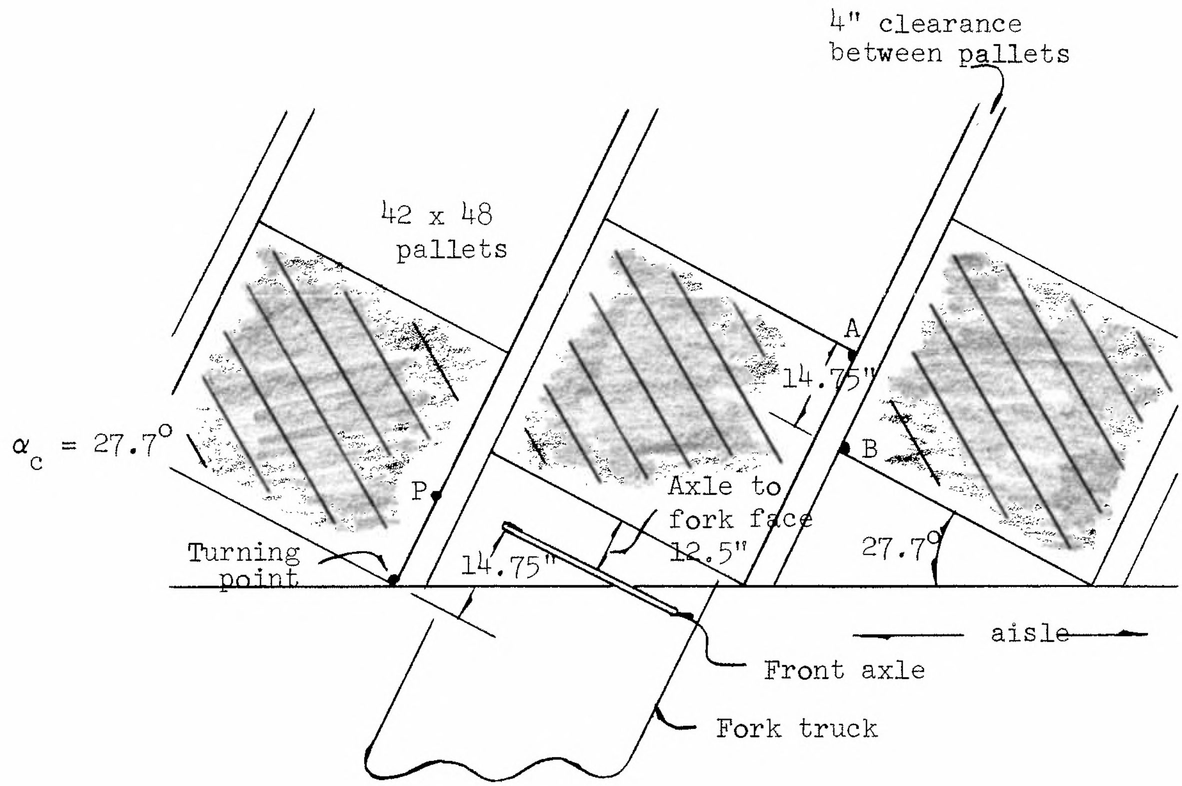


Figure 4. Pallet Removal for Critical Angle, α_c

$$AW = T.R. + (\text{pallet depth} + \text{axle to fork face}) \cos \alpha$$
$$- 2(\text{pallet width} + \text{clearance}) \sin \alpha \quad \alpha < \alpha_c \quad . \tag{2}$$

For any "slant angle" greater than α_c it will be necessary for the driver to back straight out far enough so that the pivot point of the turning radius is at the corner of the adjacent pallet, as in Figure 3. This would seem to make the minimum aisle width equal to the turning radius for any α greater than α_c. However, if α is increased enough, the rear of the fork truck will not need its full turning radius to clear the pallets on the opposite side of the aisle. An explanation of this situation is found in Appendix C (and also Appendix L). Applying this condition to the test truck's dimensions, this second critical angle (α_{c2}) is found to be 40.033 degrees. For "slant angles" greater than this critical angle, Thornton's formula No. 20[19] for angles greater than his critical angle would apply. This would be:

$$AW = T.R. [\cos(\alpha - \alpha_{c2})] + (\text{pallet depth} + \text{axle to fork face}) \cos \alpha$$
$$- 2(\text{pallet width} + \text{clearance}) \sin \alpha \quad . \tag{3}$$

Now a formula has been defined for finding the minimum aisle width at every "slant angle" considered in the study.

The aisle widths used in this study were computed using equations (1), (2), and (3), and proved to be very accurate in practice. The minimum aisle width was computed using these three equations for each angle considered in the study, and this figure, in each case was used as a base. It is necessary to provide some measure of safety clearance in every aisle.

This not only helps protect the merchandise but allows the driver to be less precise with his driving, thereby allowing the storing and picking times to be reduced. Popular practice in industry has dictated that this clearance be 2 to 6 inches. In this study the base aisle width plus the 6-inch safety clearance was used as the narrowest aisle width. Six more inches were added to this figure (a total of 12-inch clearance) to provide the driver with a liberal clearance to see if this would reduce operating times significantly. Another 6 inches was added to this figure (a total of 18-inch clearance) and tested to see if giving the driver all the room he could possibly use would cause another significant reduction in operating times. These additions to the aisle width cause a drop in space utilization, but one objective of the study was to determine whether an increase in aisle width would reduce operating times enough to offset the accompanying loss in space utilization. It would seem that anything greater than an 18-inch aisle safety clearance would reduce space utilization to the point where any increase in operating efficiency could not possibly offset the space loss. The three aisle widths used range from the minimum practical width to a width that allows the driver as much freedom as he can reasonably use. The three values allow a curve to be plotted to give some idea of the relationship between aisle width and operating times. A complete rundown on the aisle widths used (and their computation) for each "slant angle" is given in Appendix F.

In setting up the experiment, there were six angles and three aisle widths studied, giving a total of eighteen different experiments. Preliminary studies were made to determine the number of replications necessary to give a stable average time for each experiment. Allowing a short warm-up period for each experiment, it was found that fifteen good times would obtain this stable average time. An attempt was made to remove any bias from the study. One bias that could not be removed was the fact that the warehouse in which the study was made used both right angle placement and 30 degree "slant angle" placement. This made the driver more familiar with these two angles and no doubt had some effect upon his driving attitude. This bias, however, did not show up in the results of the study. The same driver was used for all of the experiments so that no changes in times resulting from driver variance would occur. Also, the same fork truck was used throughout the study. The testing sessions were all at approximately the same time of the day. This was to remove bias in the driver, who might still be sleepy early in the morning or tired late in the day. It also meant that the batteries were in about the same condition during every experiment. The experiments were all run in the same location in the warehouse to insure that the surrounding area would not have any noticeable effect on the driver's performance. During the entire study the same pallet with the same load was used to counteract any effect that different loads or pallets might have on the driver's performance. The pallets adjacent to the test area were also always the same with the same material stacked on them.

It was also not known whether there would be a different angle effect for the storing operation and the picking operation, so each experiment was divided into two categories. The first was to carry a loaded pallet in, lower it to the floor, and return to the starting area. The second category was to approach the slot unloaded, pick up a loaded pallet, and return to the starting area. Times were taken separately for each of these categories so that the total study actually consisted of thirty-six different experiments, with two performed at a time. (The order of the eighteen pairs of experiments was selected using a random number table.) The effect of the difference in times for carrying loads in and carrying loads out is included in the statistical results of the study, even though this difference has no practical value. The actual operation of the experiment is presented later on in this chapter.

The statistical methods used to evaluate the data obtained in the time studies were drawn from a text by Charles R. Hicks[13], Fundamental Concepts in the Design of Experiments. The experimental design was factorial with three factors being tested: (1) six different angles, (2) three different aisle widths, and (3) in and out, "in" being when the pallet was carried into the slot and stored, and "out" being when the pallet was already in the slot and was carried out of the area. This gave a 2 x 3 x 6 factorial design. As has been stated, the figure of fifteen replication was selected as giving an average that was stable for two decimal places in preliminary studies. The testing order of the cells was decided using a random number table.

An analysis of variance was the first test; this test will tell if there is a significant difference between any of the elements of a given factor. However, it will neither tell which element or elements are different nor will it tell by how much. The analysis of variance will also tell if there is an interaction between any of the elements, in this case, either between any two factors or between all three factors. For instance, if there were a significant interaction between angles and aisle widths, it would mean that the change in the time results for different aisle widths is not the same for each angle value. This will be further explained in the section on statistical results.

In order to find out which angles gave significantly faster or slower times and to find out which aisle widths gave significantly different results, Duncan Multiple Range Tests were run on the means obtained in the experiment. The purpose was to compare every possible pair of results within each factor and to tell whether there was a significant difference between the means. This test was very useful in deciding whether a difference in results would have any practical value. If there was no statistical difference between two results then a reliable evaluation of the monetary savings using different values of a factor (such as angles) would not be possible.

Finally, it was necessary to see if any significant amount of money could be saved on operating costs using "slant angle" pallet placement and larger aisle widths. In order to do this, monetary values had to be placed on all pertinent factors and a common unit for comparison found. The values used do not necessarily hold true in every situation. In a particular case, the proper applicable values could be inserted in the comparisons and the specific result for the case under study found. This also would be true when calculating the critical angles (α_c and α_{c2}) and aisle widths (equations (1), (2), and (3)). The main areas for which cost evaluations were needed were: (1) the labor cost for operating the fork truck; (2) other operational costs involved with running the fork truck and keeping it running (maintenance, preventative maintenance, fuel costs, machine rate per hour), and (3) the cost of floor space. The figures used in this study give some idea as to the comparisons possible in order to find the optimum compromise of angle and aisle width for both operating efficiency and space utilization.

Actual Procedure of Experiments

After deciding upon the objectives of the study, letters were sent to leading manufacturers, publishers, and users in the palletized warehousing field. All replies were to the effect that no similar study had been made and that such a study would be of value to the field. Visits were made to several local warehouses.

The Atlanta Public School Warehouse was the location selected for the experiments. The warehouse supervisor, Mr. E. W. Sammons, was most accommodating in that he provided the use of a fork truck, a fork truck driver, and a warehouse area to run the experiments. The driver, Mr. Charles Hemphill, was also very helpful. In addition to performing all of the driving tasks, he offered suggestions and opinions as the study proceeded. His opinion as a driver was that placing and picking the pallets became easier as the "slant angle" was increased. It should be pointed out that he had previous experience with right angle placement and 30 degree "slant angle" placement.

The fork truck used was two years old and had been kept in good condition. The batteries were charged every night and no testing session lasted over three hours. This was to insure that no variation in time would result from a loss in battery power.

The approach to the pallet slot and the subsequent departure from the area may be carried out in four distinct ways, as illustrated in

Figure 5, and as follows: (1) Method A consists of approaching the slot in a forward direction, turning into the slot, lowering or raising the pallet, backing out of the slot, and backing out of the area by the same route that it was entered. (2) Method B consists of approaching the slot in a forward direction, turning into the slot, lowering or raising the pallet, backing out of the slot, stopping, and then continuing on down the aisle in the original direction. (3) Method C consists of backing up the aisle from the "wrong" direction (wrong because the layout is designed for approach from the other direction), going just past the slot, stopping, moving forward into the slot, lowering or raising the pallet, backing out of the slot, and leaving the area by backing down the aisle in the original direction. (4) Method D consists of backing into the area from the "wrong" direction, going just past the slot, stopping, turning into the slot, lowering or raising the pallet, backing out of the slot, stopping, and leaving the area, moving forward, by the same route that it was entered. The desirability of these four methods is a matter of conjecture. Judging from the number of stops required in each method, it would seem that Method A would be the fastest. (A stop also implies that a change in direction was made.) Method A only requires that one change of direction be made. Methods B and C each require that two changes in direction be made. These would be the second fastest methods. Method D requires that three changes in direction be made, thereby making it the slowest. Two main factors influence the choice of which method is used. First, in a warehouse using "slant angles" every other aisle is slanted in the same direction, while the aisles in between will be slanted in the opposite direction. This means that the pallets in adjacent aisles

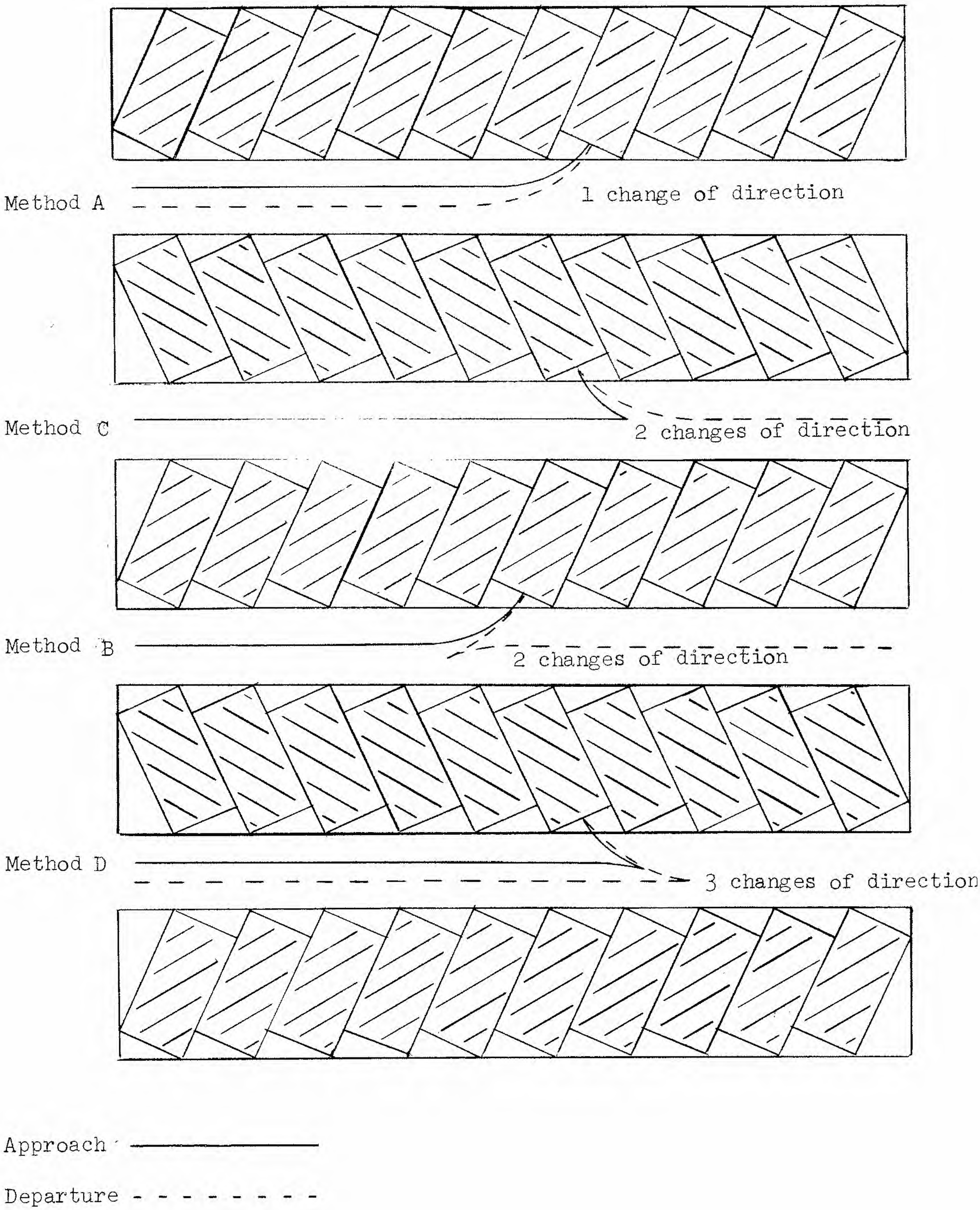
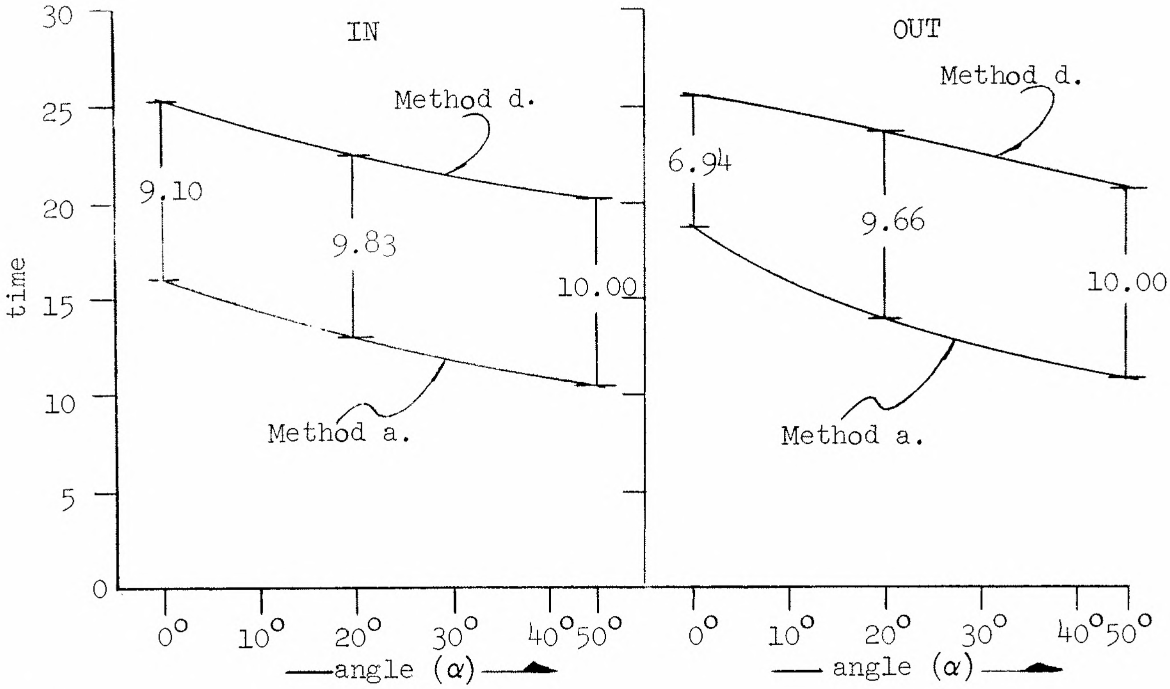


Figure 5. Methods of Approaching Storage Slot

are slanted in opposite directions. This is apparent in Figure 5. Second, the storage aisles are too narrow for two fork trucks to pass each other. This means only one fork truck in an aisle at a time. If one way traffic is used, a combination of Methods B and C will probably be used with fork trucks entering the storage aisle from a main aisle at one end and exiting into another main aisle at the opposite end of the storage aisle. This would give an average of two stops made on each trip for the placing or picking operation. If two-way traffic is allowed in each aisle, then either a combination of Methods A and D (with one main aisle) or a combination of all four methods will be used. In either case, the average number of stops will be two, so the actual time of delivery will depend on the route(s) taken to and from the pallet slot and the "slant angle" used in the warehouse. The point of discussing this subject is to show that one method is not necessarily better than the others.

In selecting a method to be used in the actual test several factors were considered. First, changes of direction consume time. The more time spent in changing direction the greater the variance of this time is going to be, so a minimum time should be spent in changing direction to reduce this variance. Second, changes of direction are fatiguing to the driver. To conserve the energy of the driver, a minimum number of changes in direction should be made. Also, if either Methods B or C were used for the tests it would mean that the driver would either have to travel around the entire bay after each delivery or pickup or back along the entire aisle to start the next cycle. This would increase the fatigue factor. With these considerations in mind, Method A was selected for its simplicity. However, to show that the method used would not be a factor, a short test run was made using Methods A and D. The results of this test are illustrated in Figure 6. The time units are arbitrarily based and the vertical difference in the times is for comparison only. It should only be construed that the vertical distance is the time difference between the two methods. However, noting that the time unit used is one hundredth of a minute (0.6 seconds), we see that the two plots are very nearly parallel, which means that the differences in the methods will be proportional. This means that a difference in method should have no effect on the relative advantages of using various slant angles.

The layout of the test area is shown in Figure 7. Point A was the pivot point for the test slot in all of the experiments. The right hand edge (line A-R) of the pallet slot was pivoted around this point according to the specific "slant angle" and the left hand line (line L-L) placed accordingly for the pallet size used. The exact pallet width was used with a 4-inch clearance on either side. Loaded pallets were placed on the edge of the clearance space at the proper angle in the proper position to help insure realistic conditions for the study. Another loaded pallet was placed directly to the rear of the test slot to insure that the pallet was placed in the same spot every time. From the computations of aisle widths the proper aisle was laid out on the floor and more loaded pallets placed along the aisle on the opposite side from the test slot. These pallets were not slanted, as this would have been inconvenient to do in the test warehouse. It was not thought that this would have any appreciable effect on the aisle width used. This is shown in Appendix L. The effects on the results, if any were present, would probably have been psychological in nature and would probably have been the same for each



*All time units on the above graphs are in 1/100ths of a minute. They are strictly arbitrary and should be used for comparison only.

Method c.

α =	0°	20°	45°
In	0.1600	0.1287	0.1013
Out	0.1866	0.1390	0.1093

Method d.

α =	0°	20°	45°
In	0.2510	0.2270	0.2013
Out	0.2560	0.2356	0.2093

These times are in minutes and should also only be used for comparison.

Figure 6. Comparison of Methods of Approach A and D

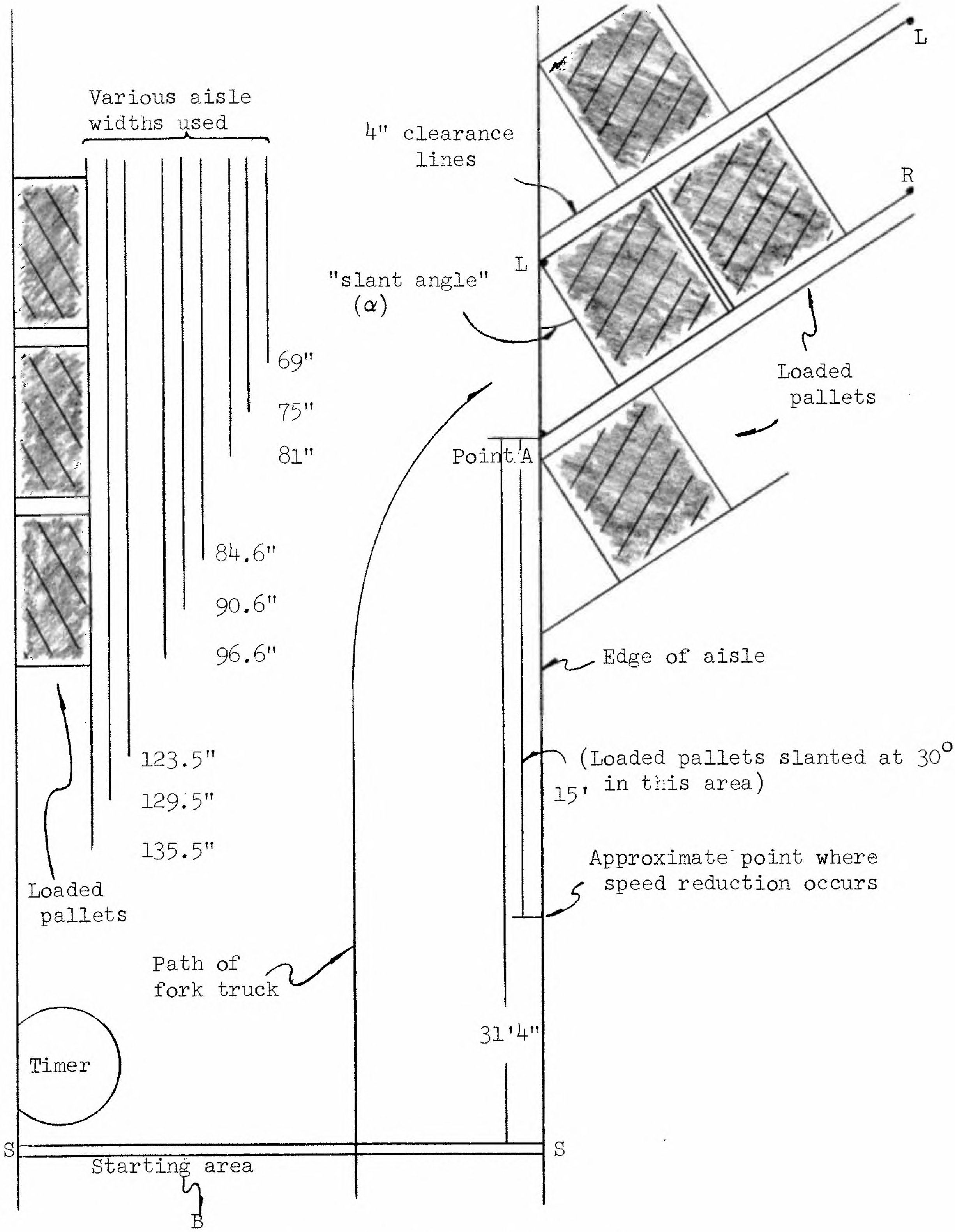


Figure 7. Test Area Layout

angle studied. These pallets were moved to the proper aisle width for each individual experiment to insure that the driver would remain within the alloted aisle width while turning in and out of the test slot.

Each run was started with the fork truck approaching from area B. Travel speed was reached before passing the starting line S-S, where timing started. The starting line was placed at a distance far enough from the test slot so that the driver did not start to slow down for his approach until he was well past the starting line. He would proceed down the aisle, lining up the fork truck in a manner appropriate for the specific angle being tested, slow down, and enter the test slot. Here he would lower the pallet, back out of the slot, and move down the aisle at travel speed until he reached the starting area, the timing being completed when he crossed the starting line. He would then repeat the operation, only coming to the slot unloaded and leaving with the loaded pallet. After a sufficient warm-up period the timing was started and continued until fifteen good times were obtained for each of the two parts of each experiment. Occasionally the driver would miss and hit an adjacent pallet. The number of occurrences of this has been noted on the experimental data table in Appendix G. These times were not included in the data for the study. The time needed to rearrange and straighten the test area was not considered to be part of this study, for most errors of this type should be eliminated through practice in the actual warehouse and a good training program. Also, such a great number of repetitions at one time was no doubt partly respsonible for these errors. Generally, no more than four or five experiments were run on any one day to keep the driver from becoming tired and biasing the results. After all of the data for the study had been gathered, the extra tests were run to determine if the method of entering the area caused an effect. The results of these are found in Figure 6. Following this part of the study, the statistical results were calculated, tabulated, and evaluated. After a graphic representation of the total times for each angle had been made, it was noted that an equation describing the relationship between times and angles might be extracted from the data. An attempt to use linear regression analysis (and the subsequent tests on the parameters found) was made to find such an equation. This will be found in Chapter III.

Finally, the statistical results were used to see if any significant money savings might be realized through the use of "slant angle" storage. This was done using "typical" monetary values placed on the various parameters. Once these had been found an attempt was made to combine the results of this study on operating efficiency using various "slant angles" and aisle widths and the previous studies made on space utilization in the warehouse. The results of the study are found in the next chapter.

CHAPTER III

DISCUSSION OF THE RESULTS

Statistical Results

A statistical evaluation of the data is important in deciding which data to trust and it helps draw reliable conclusions from the data. Used properly, the statistical results are helpful in converting what is learned from the data to an increase in the productivity of the warehouse. With properly run statistical tests it is possible to depend more on conclusions drawn from the data.

Figures 8 and 9 show graphs of the actual time data taken. Figure 8 illustrates the effect of varying the aisle width on the times for each angle, for the "in" portion and the "out" portion of the study considered separately. Figure 9 shows the effect of varying the angle on the times for each aisle width for both the "in" and the "out" portions.

The first test made was an analysis of variance. The setup of the experiment is shown in Appendix G. In the table on the first page, each cell contains three values necessary to run the analysis of variance. The top value in each cell is the mean of the fifteen data values in the cell. The second page gives the data times in seconds and presents the model equation for the ANOVA. Using the values from the first page, the resulting ANOVA table is compiled on the third page. Under "Sources" are listed the various effects to be tested. They are: (1) angle effect, (2) aisle width effect, (3) in-out effect, (4) in-out and

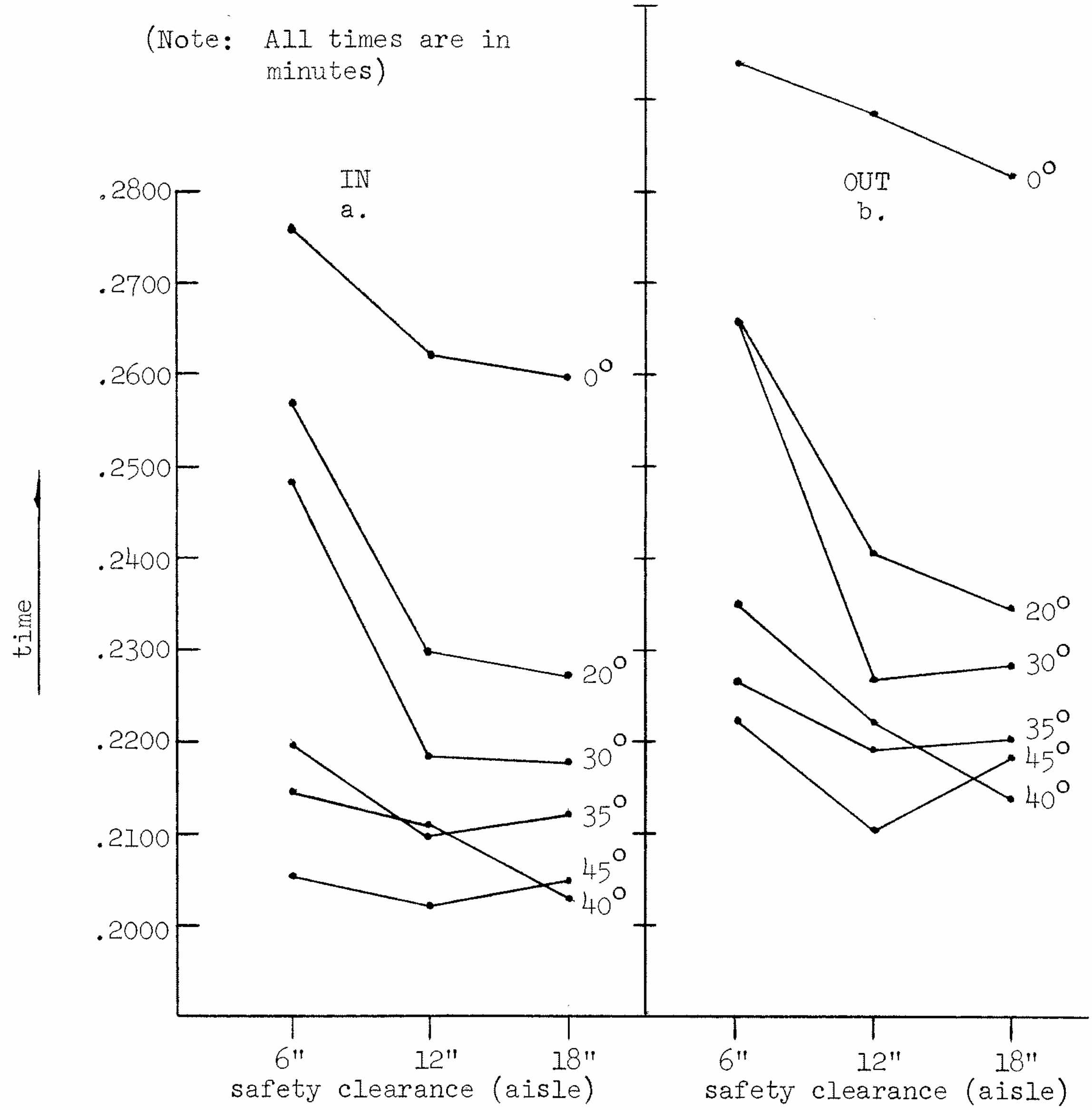


Figure 8. Time Data for Various Aisle Widths

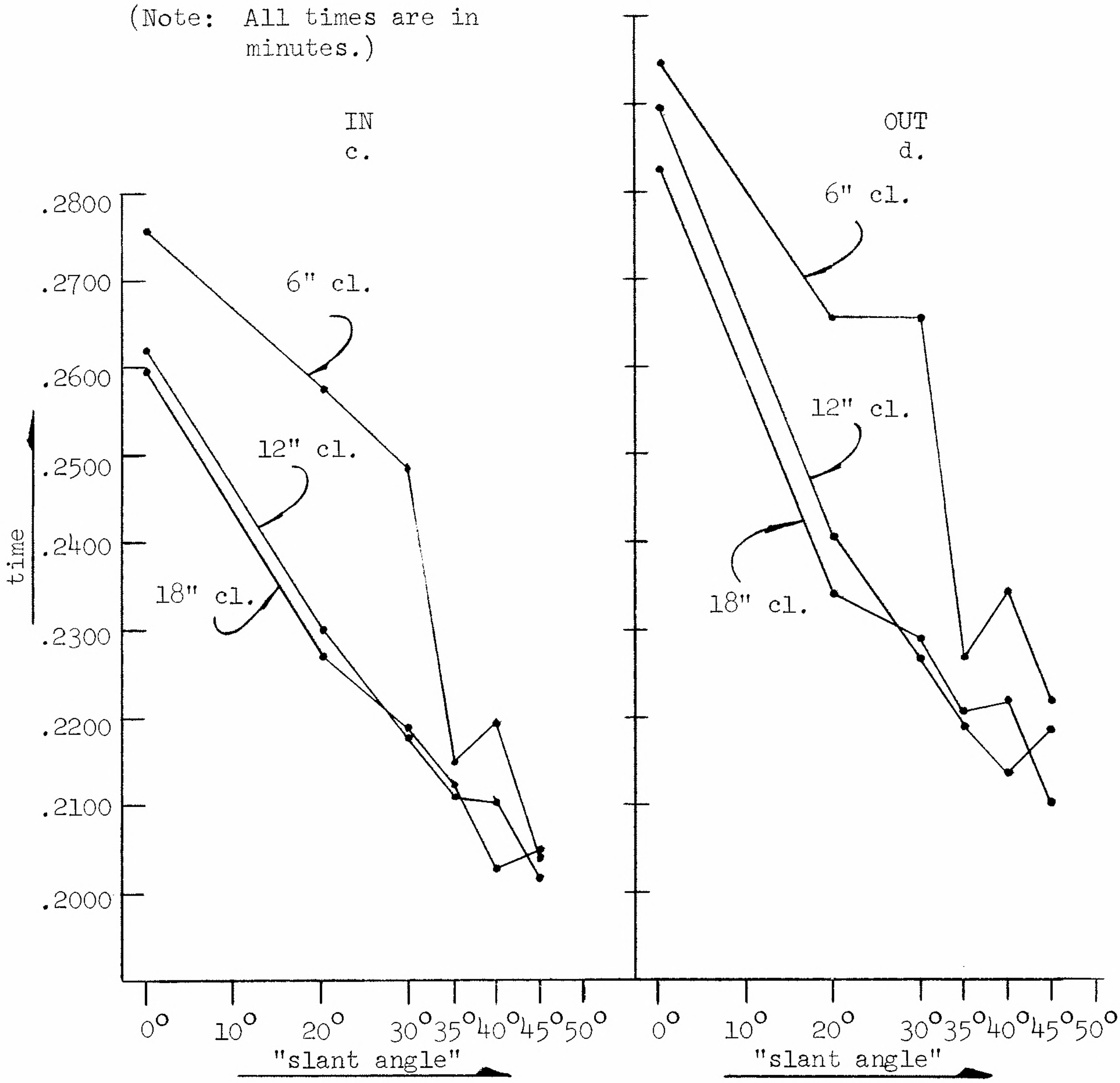


Figure 9. Time Data for Various "Slant Angles"

angle interaction effect, (5) in-out and aisle width interaction effect, (6) angle and aisle width interaction effect, (7) in-out, angle, and aisle width interaction effect. A significant result in one of the first three categories means that data values for the particular factor varied enough so that it could be concluded that this variance was not due to chance. A significant result in one of the next four sources means that there is an interaction between the two or three factors under consideration. An interaction means that the difference in response between the levels of one of the factors is not the same for all the levels of the other factor(s). In the conclusions portion of the ANOVA table we note that every one of the factors varied significantly. The high F value for the in-out effect means that there was a significant difference in the times for going in (placing) and those for going out (picking). The very high value for the angle effect means that the angle used had a very large effect on the time taken to store or remove a pallet. This supports the beliefs that prompted this study. The aisle width effect is also very significant, meaning that a change in aisle width causes a significant change in the time results. The interaction effects are significant, but are very small compared to the other effects.

This is all of the information that may be gained from the ANOVA test. Another, more helpful method of testing is applicable for the two main factors of interest: angle and aisle width.

This next method used to test the data for significant statistical results was the Duncan Multiple Range Test. This is a test designed to compare the pairs of results for a given factor to find out which pairs contain mean values significantly different from one another at a certain significance level. In order for the test to operate properly using the data already compiled in the ANOVA table, one mean must be found for each of the levels of the factor to be tested. Only one factor may be tested at a time. Two Duncan tests were run, one on the angles and another on aisle widths. The test on angles is found in Appendix H. After the necessary calculations were made and the comparisons made between every pair of angles it was evident that the test results at every angle were significantly different from each other except for the result at 35 and 40 degrees. This would seem correct looking at the plot of this data in Figure 10. This test offers definite proof that the remaining times are significantly different. After running the Duncan test and looking at the plot of the mean values, it was decided to make a linear regression analysis. This is done in Appendix H also. The correlation coefficient for the equation found is also given in the appendix. Since the correlation coefficient is very close to one, we can use this equation to find the optimal solution to the problem of combining space utilization studies with this operating efficiency study.

Next the Duncan test was run on the aisle widths. (See Appendix I.) This was to determine if there was a significant difference between the mean values for each pair of aisle widths. As one might suspect from Figure 11, there was no significant difference in the time value for the 12-inch and 18-inch safety clearance aisle widths. However, using only 6-inch safety clearance in the aisle width had significantly higher times than either of the other two. This sustained the idea that operating times and costs might be lowered by using wider aisles. It was now necessary to see if this reduction in costs was enough to offset the loss

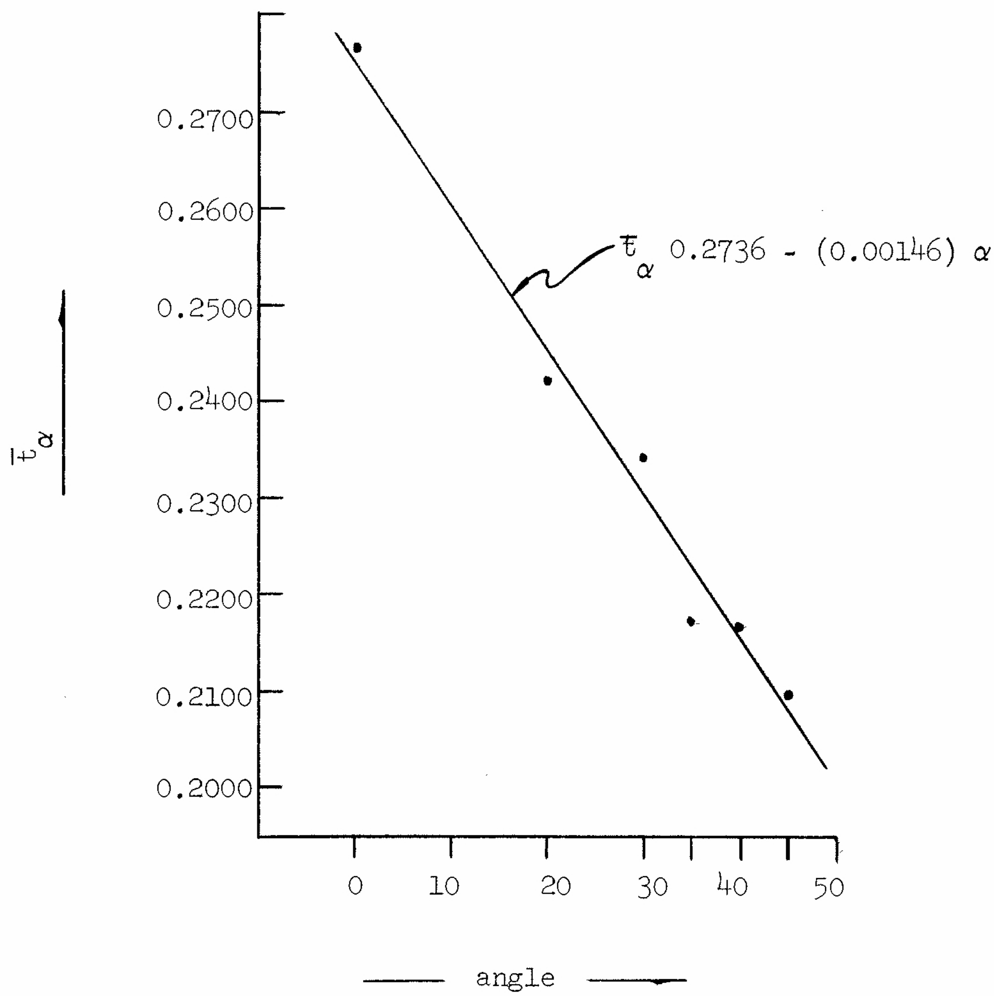


Figure 10. Graph of $\bar{t}_\alpha$ Versus "Slant Angle"

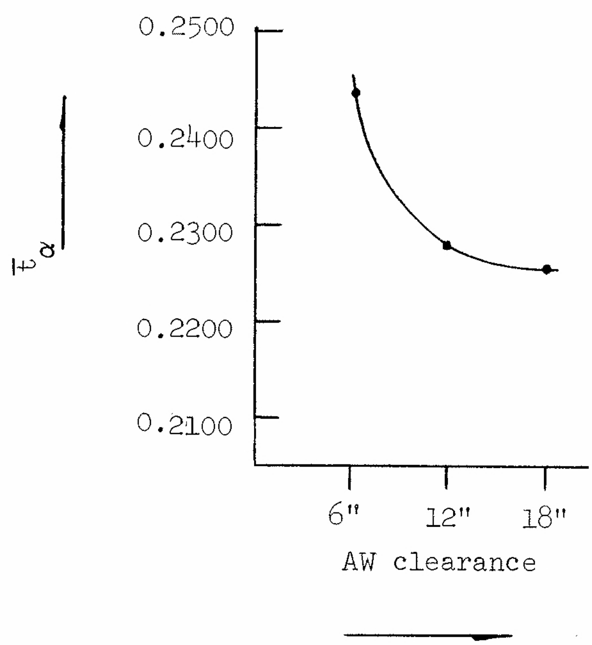


Figure 11. Graph of $\overline{t}_\alpha$ Versus AW Clearance

in space utilization and subsequent increase in floor space costs. In the event that the 12-inch and 18-inch safety clearance aisle widths are found to be economically feasible, a graph of these times and a linear regression analysis is found in Appendix H. This subject will be treated in the next section.

The statistical tests have shown that there is a significant time difference between the results at all of the angles except for 35 and 40 degrees. The statistical tests have also shown that aisle width has a significant effect on the operating times. Now, dollar values will be applied to these results to see if any significant reduction in operating costs is possible using "slant angle" pallet placement or wider aisles.

Monetary Consideration of the Results

In order to place monetary value on the results of the study, it is first necessary to make some assumptions and evaluations. Hemmi's thesis[12] is concerned with the general layout of the warehouse (size and shape of the bays). He studies space efficiency as it pertains to bay size and shape, and the travel times of the fork trucks for various layouts. These layouts do not pertain to right angle stacking only, although his studies were made using right angle stacking standard times. His times were obtained from the Eaton, Yale, and Towne Manufacturing Company[7]. Hemmi has broken time into three categories: "Base time," "Outside time," and "Inside time." "Base time" includes essentially the placing and picking operations. It starts when the driver slows to pull into a storage slot and ends when he has completed the operation and is back in the aisle under way. Hemmi also considers "Base time" to be when the fork truck is in a carrier loading or unloading, but this time was not considered in this study. The "Base time," as he uses it, is an average time for placing and removing pallets stacked two deep and three high. This average time, the "Base time," may be considered a constant as it will be the same for any given slot in a warehouse (for stacking two deep and three high). "Outside time" is the time spent outside of the building by the fork truck going to and from the carrier vehicle. This will vary with the number and location of the warehouse doors. "Outside time" was not considered in this study. "Inside time" is the time spent by the fork truck traveling between a warehouse door and a storage slot. Hemmi has found an average time to do this for eighteen different layouts. Hemmi's "Base times" and "Inside times" will be useful as a basis for making a comparison between the times for right angle stacking and "slant angle" stacking.

It was also necessary to place a value on operating costs and storage space costs. The figures used are not valid for every case, but they provided a basis for comparison. Floor space was assumed to cost $1.00/sq.ft./year. Fork truck operating costs were assumed to be $4.50/hour including operator, fuel, and all maintenance. In the example given at the end of this chapter the assumed total number of trips per year was computed considering three fork trucks capable of making 150 trips per day. This gave a total of 112,500 trips per year. A warehouse size of 6,000 pallets (2,000 pallet spaces) was used, which meant that each pallet had a turnover rate of 18.75 trips per year.

The first area investigated was the possibility that through the use of wider aisles handling times and consequently operating costs would be reduced enough to make up for the additional floor space and its associated cost. The additional aisle width was only added to the storage aisles because it was assumed that main aisles had to be wide enough for two fork trucks to pass, thereby making their width constant regardless of "slant angle" or storage aisle safety clearance. A comparison is presented in Appendix J between a bay with the normal 6-inch aisle safety clearance and a bay with a 12-inch safety clearance. The "slant angle" was varied from zero degrees to 45 degrees, and the stacking depth was varied from two deep to ten deep. The bay length was chosen to be 30 pallets. Varying this number did not change the results. In the comparison a breakeven point for each case was found. This breakeven point is the number of trips each pallet in the test bay must make in one year for the savings in operational costs to be equal to the cost of the additional floor space. If more than this number of trips is made in one year then it would be economically feasiable to widen the aisles the additional 6 inches. The results of this comparison have been ploted in Figure 12.

Looking at these graphs it is possible to decide whether it is economically feasible to increase the aisle width if the angle of stacking, depth of stacking, and number of trips per year per pallet are known. The figures used in obtaining these graphs may be found in Appendix J. It should also be remembered that these figures are based on an operating cost of $4.50/hour and a floor space cost of $1.00/sq.ft./year. A change in these cost figures will produce different breakeven points. In general, it is thought that an extremely large turnover rate per pallet is necessary

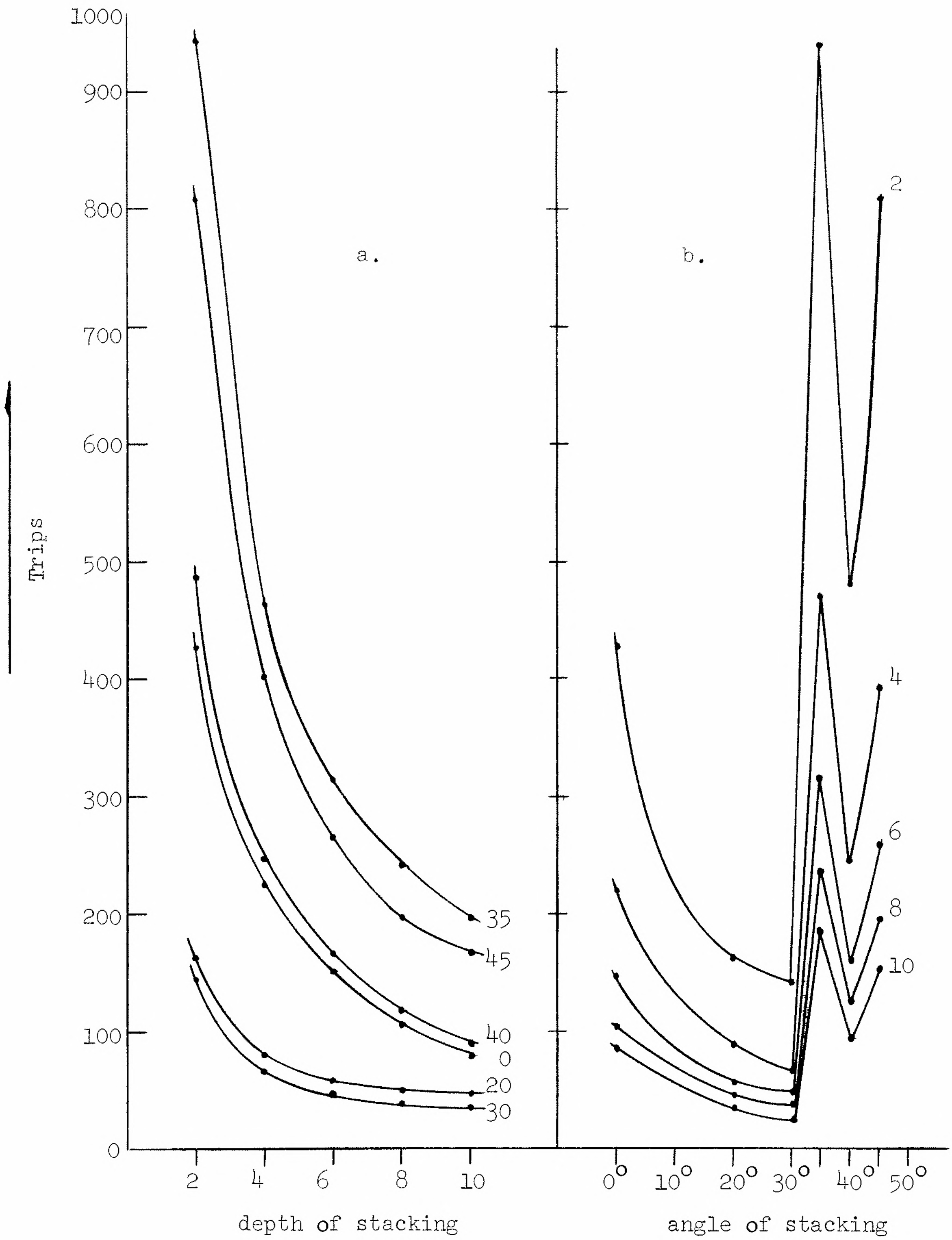


Figure 12. Breakeven Points for Additional Aisle Clearance

to realize any savings in this area. The minimum turnover rate encountered in the comparison made was 28 trips/pallet/year. For a warehouse of 6,000 pallets this means 168,000 trips/year or 672 trips/day. This would require at least four fork trucks working constantly. Judging from these results, it does not appear to be economically feasible to widen the aisles to lower handling times. A study should be made for each individual case and its particular cost figures if it is thought that the turnover rate of each pallet is large enough to warrant a study of this nature. This is done for the example at the end of the chapter.

The second area to be investigated is a comparison between "slant angle" storage times and "right angle" storage times. In all cases a "trip" is considered to be an average round trip from warehouse door to storage space to warehouse door. The associated time will be an average of the times for carrying material into the warehouse and carrying material out of the warehouse.

Table 1 gives a direct comparison between right angle picking and "slant angle" picking for stacking one deep and one high. The times found in the actual time study have been corrected for this table so they reflect only the picking or placing time. These times are found in column two. Column three shows the average time savings per trip. Column four shows the percentage of time saved by using the particular "slant angle" as opposed to using right angle placement. Column five shows the dollar savings to be realized per trip by using the particular "slant angle." The time saved will be constant regardless of the total length of the individual trip, so the (total operational) savings to be realized in a warehouse will be directly proportional to the number of

Table 1. Comparison of "Slant Angle" Time with Right Angle Time

Angle	Average Time Per Trip (Minutes)	Average Time Saving Per Trip (Minutes)	Percent of Time Saved Per Trip*	Operating Cost Dollar Savings Per Trip
0	0.2476	---	---	---
20	0.2137	0.0339	13.7	$ .00254
30	0.2057	0.0419	18.6	.00314
35	0.1891	0.0585	23.6	.00438
40	0.1889	0.0587	23.7	.00440
45	0.1823	0.0653	26.4	.00490

*Compared to right angle placement.

Table 2. Comparison of "Slant Angle" "Base Time"* with Right Angle "Base Time"

Angle	Average Time Per Trip (Minutes)	Average Time Saving Per Trip (Minutes)	Percent of Time Saved Per Trip**	Operating Cost Dollar Savings Per 150 Trips
0	0.650	---	---	---
20	0.616	0.0339	5.23	$ .382
30	0.608	0.0419	6.47	.427
35	0.592	0.0585	8.93	.657
40	0.591	0.0587	9.08	.660
45	0.585	0.0653	10.00	.735

*From Hemmi[12].

**Compared to right angle placement.

trips that are made in the particular warehouse in a certain period of time. The total savings for a period of a year can be calculated by taking the total number of trips for all of the fork trucks into and out of a "slant angle" storage area and multiplying this times the dollar savings per trip for the particular "slant angle" used. This will be done in the example at the end of the chapter.

Table 2 shows a direct comparison between the "base time" for right angle stacking and the "base times" for various "slant angles." This "base time" is the picking or placing time for stacking three pallets high and two pallets deep. This particular configuration is used so that the times found in Hemmi's thesis[12] may be used as a basis for comparison. In the table, column two shows the actual average time taken for the operation. Column three shows the time savings to be realized per trip. Column four shows the percentage of time saved by using a particular "slant angle" as opposed to using right angle placement. Column five shows what the typical daily savings for one fork truck would be for each angle if the fork truck were to make 150 trips per day (150 was the number of trips made per day for each fork truck in a beer warehouse study). From these figures it is seen that while operational costs will be reduced (by using "slant angle" storage), the reduction will probably not be a highly significant amount.

For further comparisons, it is necessary to consider a particular layout. Hemmi, in his thesis[12], tested a number of layouts for their operating efficiencies at various capacity levels, namely 234, 1000, and 2000 pallet floor spaces. Figure 13 shows the layout that was the most efficient at the 2,000 pallet space capacity level. This layout was made

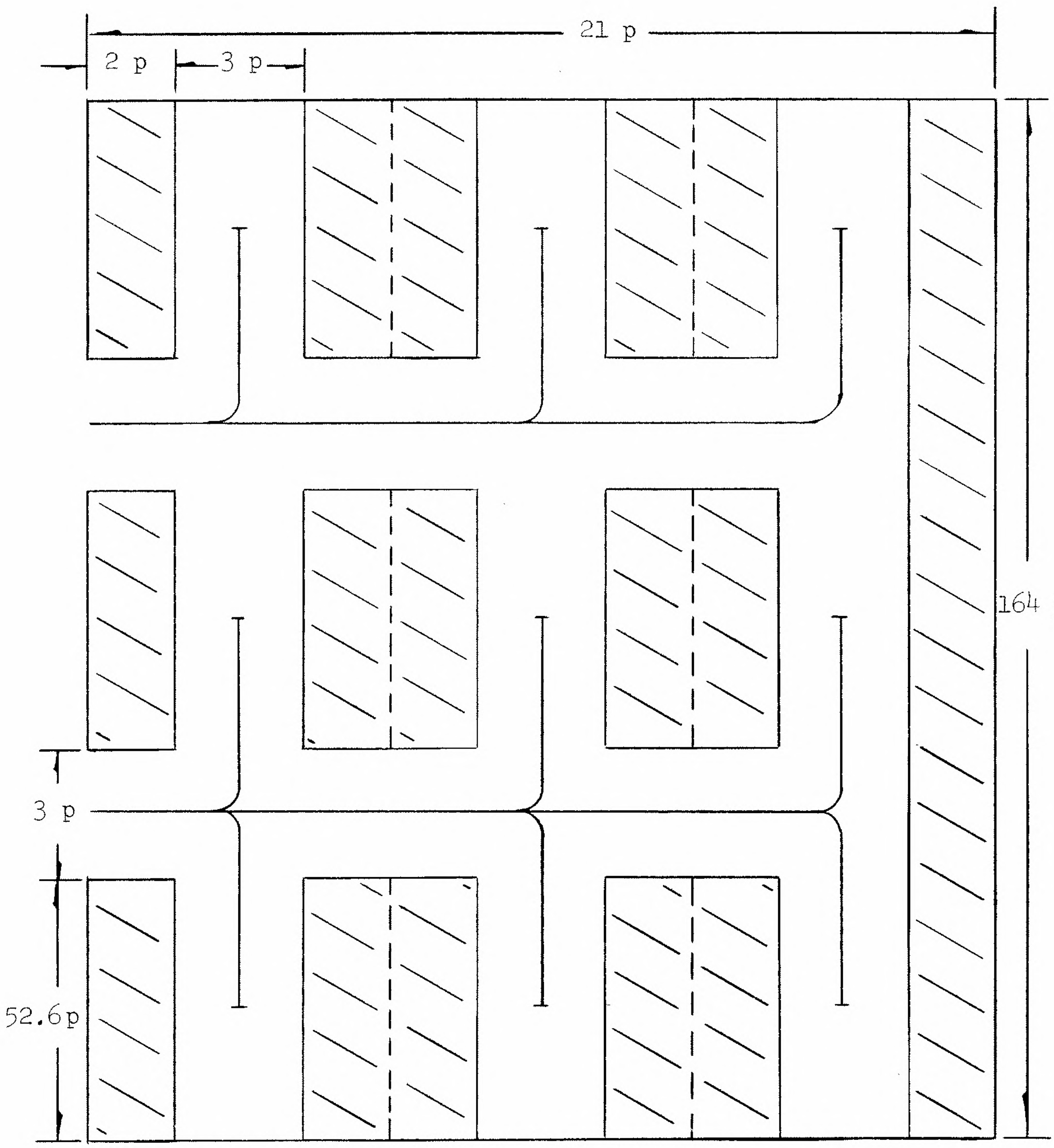


Figure 13. Hemmi's Layout No. 17

extremely long and narrow when it was expanded from its original 234 pallet size to the 2,000 pallet size. (Figure 13 is not to scale so this is not readily evident.) This resulted from the fact that the lay-out was expanded in only two directions (one dimension). This layout will, however, serve as a basis for comparison. Table 3 was tabulated considering "inside time" and "base time" combined for this particular layout. Column two shows the average total time inside the warehouse per trip for this layout. The time outside of the warehouse will vary according to the carrier, dock facilities, and unloading method, and was not considered. Column three shows the average time saved per trip. Column four shows the percentage of time saved by using the particular "slant angle" as opposed to using right angle placement. Column five shows the dollar savings per pallet space for one year for this particular layout where the pallets are stacked three high. The turnover rate was assumed to be thirty trips per space or ten trips per pallet in one year. These figures also indicate that the dollar savings to be realized by reducing operating costs will not be of a highly significant amount. Figure 14 gives a graphic illustration of column five in Table 3. This illustrates that operating cost savings increase linearly as the "slant angle" is increased.

Figure 15 is useful for finding the annual operating cost savings possible for a certain number of trips per year using a certain "slant angle." The x-axis gives the trunover rate in both trips per day and trips per year. By using the lines plotted for each "slant angle" the annual savings may be found on the y-axis. This graph will be used in the following example.

Table 3. Money Savings Considering "Inside Time"*

Angle	Min. Time Per Trip (Minutes)	Time Savings Per Trip (Minutes)	Percent of Time Saved Per Trip	Dollar Savings Per Trip Per Pallet Space
0	1.510	---	---	---
20	1.476	0.034	2.25	$ 0.0765
30	1.468	0.042	2.78	0.0945
35	1.452	0.058	3.84	0.1305
40	1.451	0.059	3.90	0.1328
45	1.445	0.065	4.30	0.1462

*"Inside time"--(from Hemmi[12]) has been modified to include "Base time," also. "Base time" is 0.650 minutes. Times are for warehouse of 2000 pallet spaces, stacked two deep and three high. Frequency is ten trips/year/pallet.

Table 4. Total Dollar Savings per Year for Operating Costs and Space Costs for Sample Case

Angle	Area* Per Pallet Space (Sq. Ft.)	Space Savings Per Pallet Space (Sq. Ft.)	Money Savings for Space Utilization Per Year	Money Savings for Operating Efficiency Per Year	Total Dollar Savings Per Year
0	29.48	---	---	---	---
5	29.48	0.00	$ 0.00		
10	29.36	0.12	240.00		
15	29.43	0.05	100.00		
20	29.30	0.18	360.00	$ 283.00	$ 643.00
25	29.55	-0.07	-140.00		
30	29.65	-0.17	-340.00	350.00	10.00
35	30.14	-0.66	-1320.00	490.00	-830.00
40	23.80	5.68	11360.00	500.00	11860.00
45	24.77	4.71	9420.00	550.00	9970.00
50	25.99	3.49	6980.00		

*From Ballou[4].

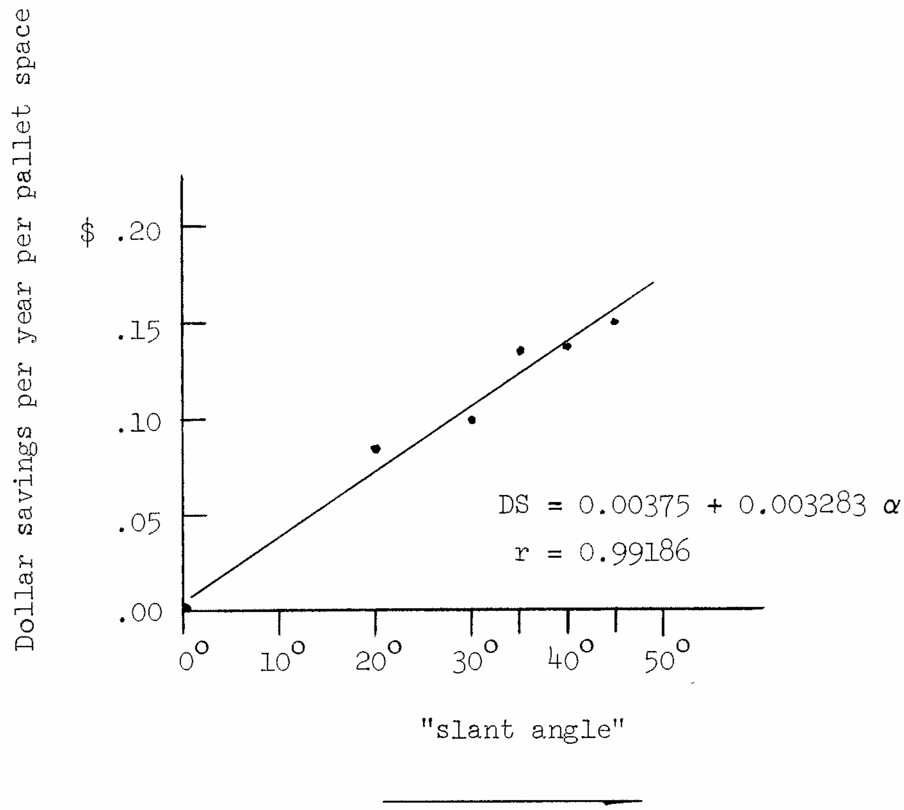


Figure 14. Money Savings Per Pallet Space Per Year Versus "Slant Angle" (from Table 3)

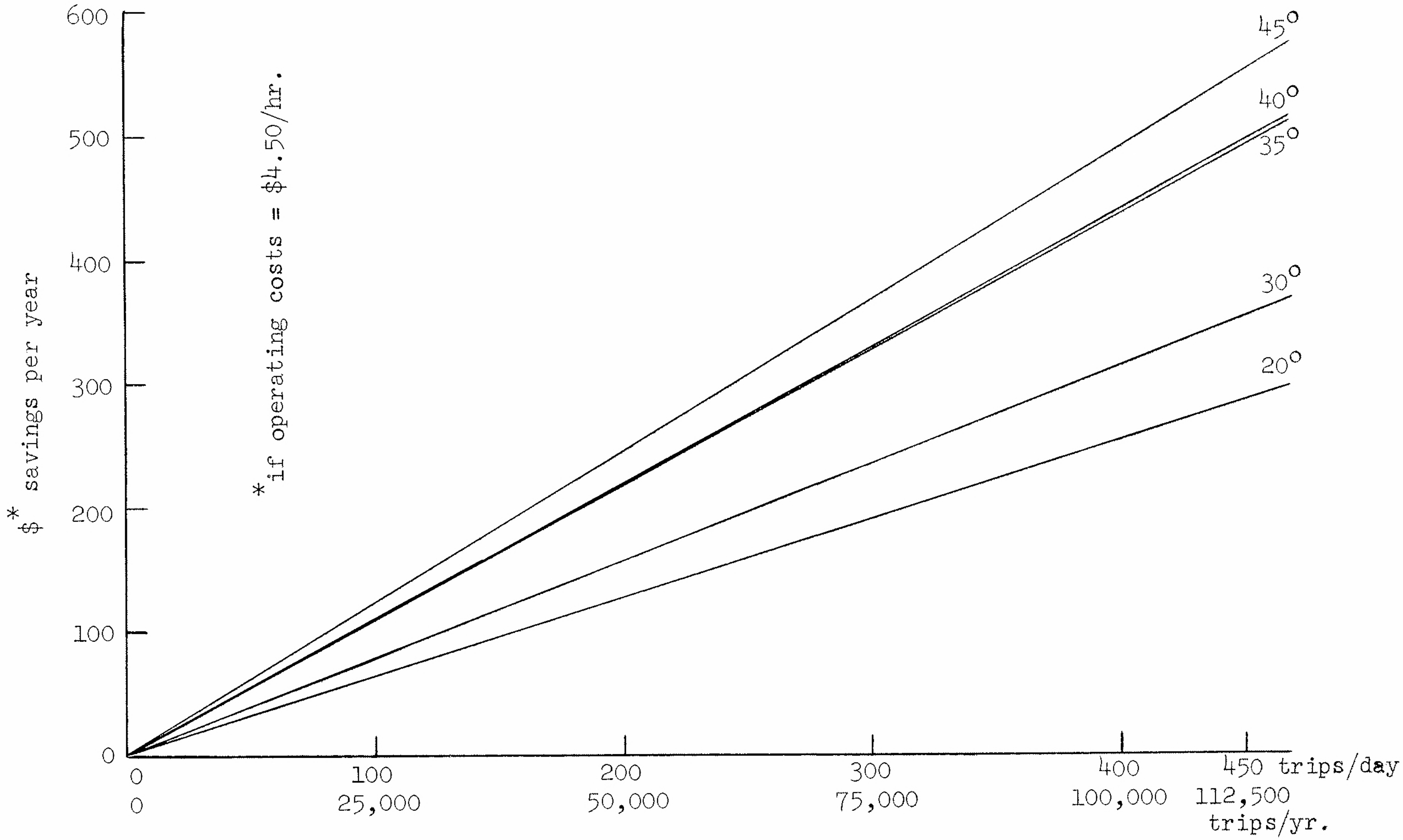


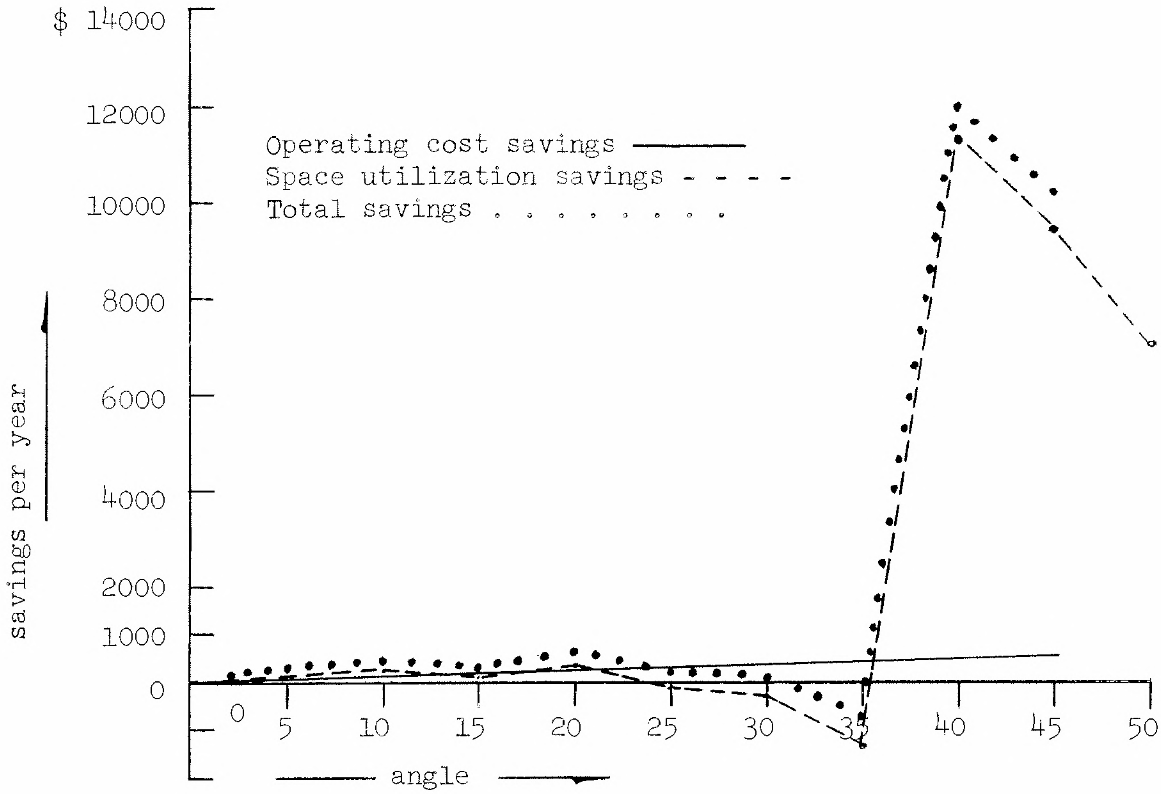
Figure 15. Operating Cost Savings

Example:

In order to clarify how to figure the amount of money that may be saved on operational costs and to show how the resultant savings from both this study and a space utilization study may be combined, an example is given and the resultant savings shown. The warehouse layout used is the one in Figure 13 from Hemmi[12]. The pallets are to be stacked three high and two deep. The size of the warehouse is 2,000 pallet spaces or 6,000 pallets. Three fork trucks are used and they make 150 trips per day per fork truck. There are 250 normal working days in a year which will give a total of 112,500 trips per year for the warehouse total. This gives a turnover rate for each pallet of 18.75 trips per year. Checking Figure 12 and Appendix K, it is evident that no attention need be given to the possibility of using wider aisles to increase operating efficiency and lower total costs. Using Figure 15, the operating cost reductions were found and recorded in column five of Table 4. These figures were found by finding the 450 trips per day mark on the x-axis, then going to the appropriate line for each angle, and find the annual amount saved on the y-axis. Using the form found in Ballou's article[4] the area required per pallet space in the sample warehouse was found. These figures are found in column two of Table 4. Column three shows the difference between the space required for one pallet at each "slant angle" and the space required for one pallet at a right angle. This figure is expressed as a savings over the right angle placement. Using a space value of $1.00/sq.ft./year, column four shows the floor space utilization savings possible per year for each of the "slant angles" tested. Column five shows the operating cost savings for each "slant angle."

Column six shows the total savings to be realized at each "slant angle." Figure 16 is a graph of columns four, five, and six in Table 4, showing the space utilization dollar savings, the operating efficiency dollar savings, and the total dollar savings possible for this particular layout and turnover rate. It is evident from both Table 4 and Figure 16 that while operating costs may be reduced a few hundred dollars per year, the great majority of cost reduction through the use of "slant angle" storage comes from space utilization savings. This is evidenced by the optimum solution of the example. An angle of 40 degrees shows the maximum savings. At this angle the space utilization dollar savings are $11,360 and the operating cost dollar savings are only $500.

From these results it can be seen that the most important area to be considered in choosing a "slant angle" is that of space utilization. The exception to this would be if a small palletized warehouse had an extremely high turnover rate. For such a case, the operating cost savings might be enough to affect the angle chosen using a space efficiency study. Also, in such a case, a larger aisle width might be feasible according to Figure 12, which would consequently affect the space efficiency study.



Angle	Space Utilization Dollar Savings Per Year	Operating Cost Dollar Savings Per Year	Total Dollar Savings Per Year
0	---	---	---
5	$ 0.00		
10	240.00		
15	100.00		
20	360.00	$ 283.00	$ 643.00
25	-140.00		
30	-340.00	350.00	10.00
35	-1320.00	490.00	-830.00
40	11360.00	500.00	11860.00
45	9420.00	550.00	9970.00
50	6980.00		

Figure 16. Savings Per Year
(See page 53 for example)

CHAPTER IV

CONCLUSIONS

This study has proven that the operating times for "slant angle" pallet arrangements are significantly less than the times for right angle pallet arrangements. Evaluating this fact in cost savings has shown that the amount of money saved is only a few hundred dollars per year for a "typical" case, the "typical" case being a 6,000 pallet warehouse with 112,500 trips made per year.

When considering operating cost reduction there is no optimum angle to be found. The study has shown that a graph of operating time taken as a function of the "slant angle" is linear in form and decreases as the "slant angle" is increased. Therefore, to reduce operating costs as much as possible, the greatest "slant angle" that is permissible under other considerations, primarily space utilization, should be used.

The study has proven that widening the aisles to reduce operating times and operating costs is not justified economically because the value of the floor space lost is greater than the money savings realized through the lower times. The exception to this would be a small warehouse operation with a high turnover rate per pallet. In such a case a comparison, as in Appendix K, should be made.

A joint study, employing a space utilization study and the results of this operating efficiency study was made on a sample warehouse. The space utilization study was from Ballou[4] and the warehouse layout used was from Hemmi[12]. The results of this joint study show that the space utilization study will save the warehouse operation a far greater amount of money. In the sample study made, the space utilization money savings were $11,360 per year, while the operating cost savings were only $500 per year. These values were for the optimum solution of the sample study. The conclusion has to be that if limited funds are available a space efficiency study would be of far greater value to the "typical" larger warehouse operation. In an unusual situation where the turnover rate per pallet is exceptionally high, the operating cost reduction study will be significant.

In conclusion, it can be stated that "slant angle" pallet placement has a large number of advantages over right angle pallet placement. The primary advantage is the large annual savings to be realized through better floor space utilization. The second most important advantage is the reduced operating costs. Other advantages are: (1) less corner damage <u>during the placing and picking operation</u>, (2) easier location of items for moving and inventory purposes, (3) less driver fatigue because of less turning, and (4) easier and more accurate lining up on pallet slot. Right angle placement should be used for bays that are along a wall (or in short bays that butt into walls) allowing access only from one storage aisle. This is the case studied by Thornton in his master's thesis[19], and he has proven that right angle placement is best for maximum space utilization in this case. Other advantages of right angle placement are less corner damage <u>during aisle travel</u> and two-way traffic in storage aisles. The conclusion of this study must be that "slant angle" pallet placement, with the necessary investigation to find the optimal angle, is, both economically and otherwise advantageous over right angle placement for storage in a modern palletized warehouse.

CHAPTER V

RECOMMENDATIONS

There are other areas in this field where additional study should increase warehouse efficiency still more. A study should be made to find a method for computing the optimal bay size for a certain pallet size, truck size, and accessibility requirement. The stacking height, size, and capacity should also be considered in this study. The "slant angle" would also have to be considered because it would affect the outside shape of the bays.

A study could be made in order to write a computer program that would combine the results of a space efficiency study, an operating efficiency study, and a bay size study, with the requirements and limitations of a specific warehouse. The final output would be the specifications for the appropriate warehouse design.

An area which storage rack manufacturers should look into is that of "slant angle" storage racks. Since such significant savings are possible with regular "slant angle" placement, these same savings should be realized by using "slant angle" racks. They would probably require additional capital investment but would no doubt result in significant dollar savings over conventional right angle racks.

Perhaps in the distant future all warehouses will be totally automatic, using stackers, conveyors, and other mechanical devices, and being run by computers, and fork trucks and men will not be needed.

Until then, however, with the great variety of shapes, weights, and sizes of goods to be stored, the fork truck and the pallet will have their place and need to be studied.

APPENDIX A

62

FE 30

3,000 lb. LIFT TRUCK . . . CUSHION TIRES

OPERATING DATA

Capacity	3000 lbs.
Load Center	24 inch
Inch Pound Rating (Mast Vertical)	109,500 in. lbs.
Right Angle Stacking Aisle (add Load Length)	75½ inches
Turns in Intersecting Aisles	57 inches

TRAVEL SPEEDS @ 36 VOLTS

Empty . . . 6.5 mph   Loaded . . . 6.0 mph

LIFT SPEEDS @ 36 VOLTS

Loaded	50 fpm
Empty	65 fpm

TREAD

Drive Wheels	28 in.
Steer Wheels	28 in.

TIRES (Standard)

Drive Tire Size — Cushion	18 x 6 x 12⅛
Steer Tire Size — Cushion	13½ x 4½ x 8

EXTRA LIFT MAST DIMENSIONS

	Stack Hgt.	Free Lift	Free Lift Hgt.	Low'd Hgt.	Ext. Hgt.
Lift 106″	107¾″	15″	16¾″	71″	127¼″
Lift 130″	131¾″	15″	16¾″	83″	151¼″
Lift 144″	145¾″	15″	16¾″	90″	165¼″

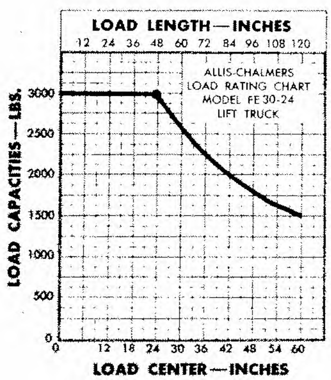
POWER SUPPLY

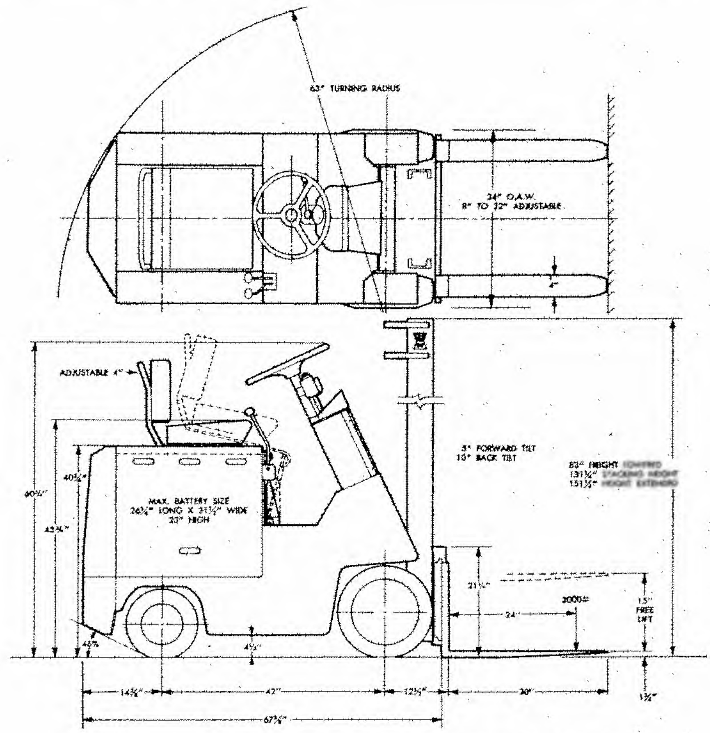
Lead acid battery in steel tray having adequate kilowatt hour capacity. Batteries exceeding standard compartment width require steel tray with cover. Open type compartment with protective rubrails will be furnished.

18 cells of 13 to 21 plate—standard
16 cells of 13 to 21 plate—optional
15 cells of 13 to 21 plate—optional

WEIGHT

With Battery (Approx) . . . 5900 lbs.





The Allis-Chalmers Mfg. Co. reserves the right to make changes in the specifications or to add improvements at any time without notice or obligation.

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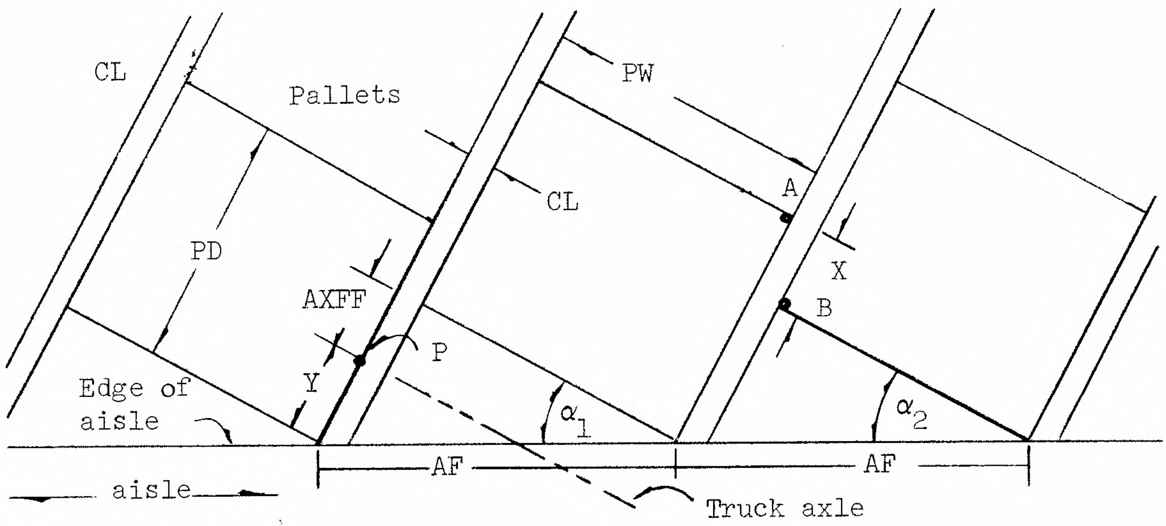
APPENDIX B

Derivation of First Critical Angle (α_c) for Application of Aisle Width Formulas



To find critical angle (α_c), assume:

(1) $\alpha_1 = \alpha_2 = \alpha_c$

(2) $Y = X$

(3) PW > truck width

PD = Pallet Depth

PW = Pallet Width

α = "Slant Angle"

CL = Clearance Btwn Pallets

AXFF = Distance from Axle to Fork Face

AF = Distance Pallet Faces on Aisle

From diagram:

1) $\text{SIN } \alpha_1 = \dfrac{Y + AXFF}{AF}$

2) $\text{SIN } \alpha_2 = \dfrac{PD - X}{AF}$

3) $Y + AXFF = PD - X$ (from 1 and 2)

4) $2Y = PD - AXFF$ (since $X = Y$)

5) $\text{TAN } \alpha_1 = \dfrac{Y + AXFF}{PW + CL}$ (from diagram)

6) $Y = \text{TAN}\ \alpha_1(\text{PW} + \text{CL}) - \text{AXFF}$ (from 5)

7) Substituting (6) in (4)

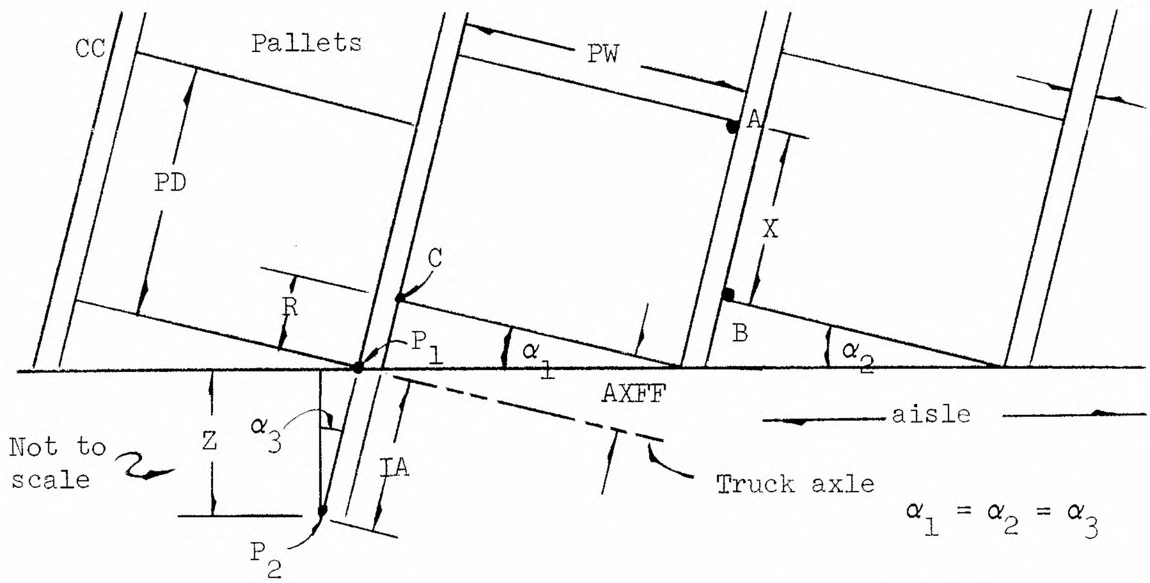
$$2[\text{TAN}\ \alpha_1(\text{PW} + \text{CL}) - \text{AXFF}] = \text{PD} - \text{AXFF}$$

8) $2\ \text{TAN}\ \alpha_1(\text{PW} + \text{CL}) - 2\ \text{AXFF} = \text{PD} - \text{AXFF}$

9) $\text{TAN}\ \alpha_1 = \dfrac{\text{PD} + \text{AXFF}}{2(\text{PW} + \text{CL})}$ (since $\alpha_c = \alpha_1$)

10) $\alpha_c = \text{TAN}^{-1}\left(\dfrac{\text{PD} + \text{AXFF}}{2(\text{PW} + \text{CL})}\right)$

Derivation of Minimum Aisle Width Formula for $\alpha < \alpha_c$
(see Appendix C for large α)



To find min. aisle width for $\alpha (\alpha < \alpha_c)$:

1) $\text{TAN } \alpha_2 = \dfrac{PD - X}{PW + D}$

2) $X = PD - \text{TAN } \alpha_2 (PW + D)$

3) $\text{TAN } \alpha_1 = \dfrac{R}{PW + CL}$

4) $R = \text{TAN } \alpha_1 (PW + CL) = X - Y + AXFF$

5) $IA = X - R + AXFF$

(what this amounts to is the difference between X and Y in the previous derivation)

6) $\text{COS } \alpha_3 = \dfrac{Z}{IA}$

7) $Z = \text{COS } \alpha_3 (X - R + AXFF)$

PD = Pallet Depth

PW = Pallet Width

α = "Slant Angle"

CL = Clearance Between Pallets

AXFF = Distance from Axle to Fork Face

X = Distance for Pts. A & B to Clear

R = Distance from Pt. C to Edge of Aisle

IA = Distance Truck Axle Must Travel Into Aisle to Make Turn

Z = Perpendicular Dist. into Aisle (from IA)

8) $Z = \cos\alpha_3 \left[(PD - \tan\alpha_2[PW + CL]) - (\tan\alpha_1[PW + CL] + AXFF \right]$

9) $Z = \cos\alpha_3(PD + AXFF) - 2\sin\alpha_3(PW + CL)$

10) AW min. $= Z +$ T.R. for $\alpha < \alpha_c$

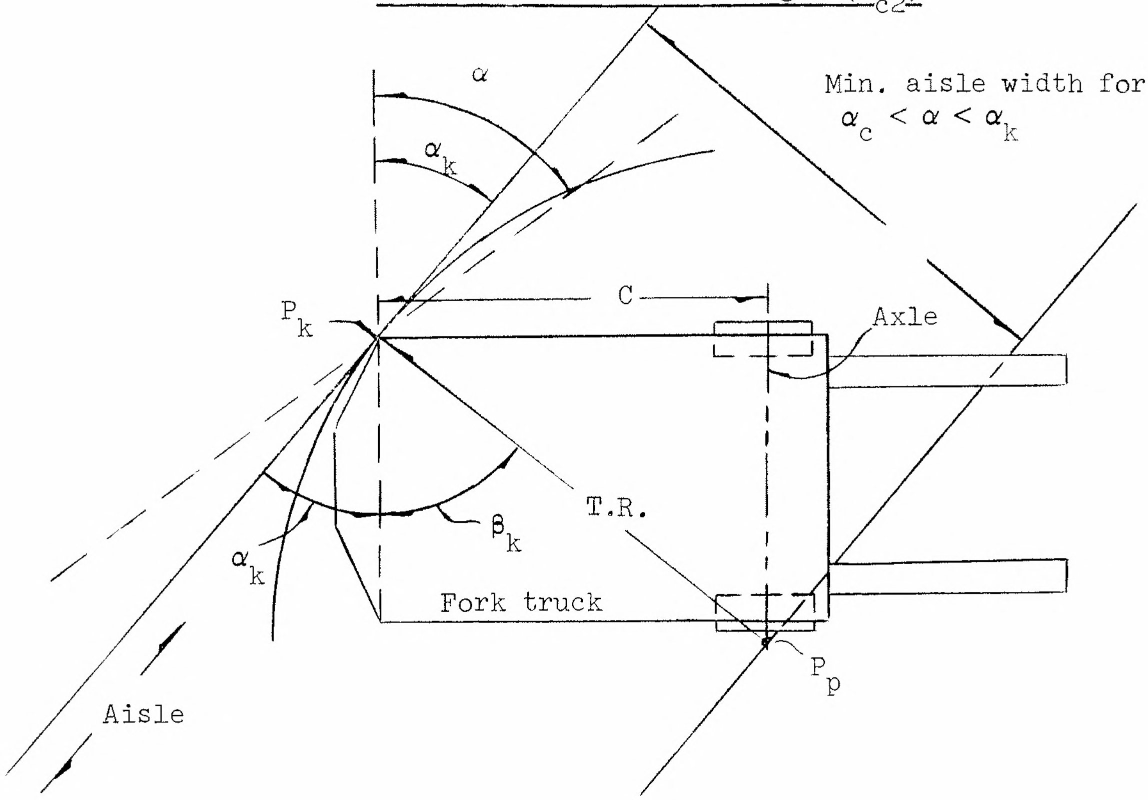
AW min. = Minimum Aisle Width

P_1 = Min. Pivot Point

P_2 = Pivot Point for $\alpha < \alpha_c$

APPENDIX C

Derivation of Critical Angle (α_{c2})



The turning radius is determined by a critical point on the rear of the fork truck. In order to reduce the aisle width to less than the T.R., α must be greater than α_k. α_k is found by constructing a tangent (at the critical point) to the T.R. Knowing the perpendicular distance from P_p to P_k, we may find α_k.

$$\text{SIN } \beta_k = \frac{c}{\text{T.R.}}$$

$$\beta_k = \text{SIN}^{-1} \frac{c}{\text{T.R.}}$$

since $\alpha_k + \beta_k = 90^\circ$:

$$\alpha_k = \text{COS}^{-1} \frac{c}{\text{T.R.}}$$

which validates Thornton's[9] formula for aisle width for $\alpha > \alpha_k$. In the test case in this study $\alpha_k = 40^\circ$ Z'.

APPENDIX D

Calculation of Sums of Squares for Anova Table

$$\Sigma X = 125.17$$

$$(\Sigma X)^2 = 15,667.5289$$

$$\Sigma X^2 = 29.380000$$

$$SS_{total} = 29.380000 - \frac{(125.17)^2}{540} - \frac{15,667.5289}{540} - 29.013942 = .366058$$

$$SS_{in\text{-}out} = \frac{(60.970)^2 + (64.2)^2}{270} - 29.013942 = .019321$$

$$SS_{angle} = \frac{(24.705)^2 + (21.69)^2 + (20.975)^2 + (19.475)^2 + (19.46)^2 + (18.865)^2}{90}$$

$$- 29.01394 = .259382$$

$$SS_{aw} = \frac{(43.74)^2 + (40.9)^2 + (40.53)^2}{180} - 29.013942 = .034372$$

$$SS_{in\text{-}out \times angle} = \frac{(11.935)^2 + (10.655)^2 + (10.230)^2 + (9.54)^2}{45}$$

$$+ \frac{(9.460)^2 + (9.150)^2 + (12.770)^2 + (11.035)^2}{45}$$

$$+ \frac{(10.745)^2 + (9.935)^2 + (10.000)^2 + (9.715)^2}{45}$$

$$- (.186945) - (.019321) - (29.013942)$$

$$= .073935$$

$$SS_{\text{in-out x aw}} = \frac{(21.265)^2 + (19.795)^2 + (19.795)^2 + (22.475)^2}{90}$$

$$+ \frac{(20.99)^2 + (20.735)^2}{90} - (.019321) - (.034272)$$

$$- 29.013942 = .000202$$

$$SS_{\text{angle x aw}} = \frac{(.854)^2 + (.779)^2 + (.766)^2 + (.659)^2 + (.678)^2}{30}$$

$$+ \frac{(.638)^2 + (.7015)^2 + (.6645)^2 + (.643)^2}{30}$$

$$+ \frac{(.645)^2 + (.820)^2 + (.616)^2 + (.7965)^2 + (.6885)^2}{30}$$

$$+ \frac{(.667)^2 + (.6455)^2 + (.623)^2 + (.6325)^2}{30}$$

$$- (.259382) - (.034272) - (29.013942) = .016082$$

$$SS_{\text{aw x angle x in-out}} = \frac{440.19}{15} - .019321 - .259382 - .034272 - .001498$$

$$- .000202 - .016082 - 29.013942 = .001301$$

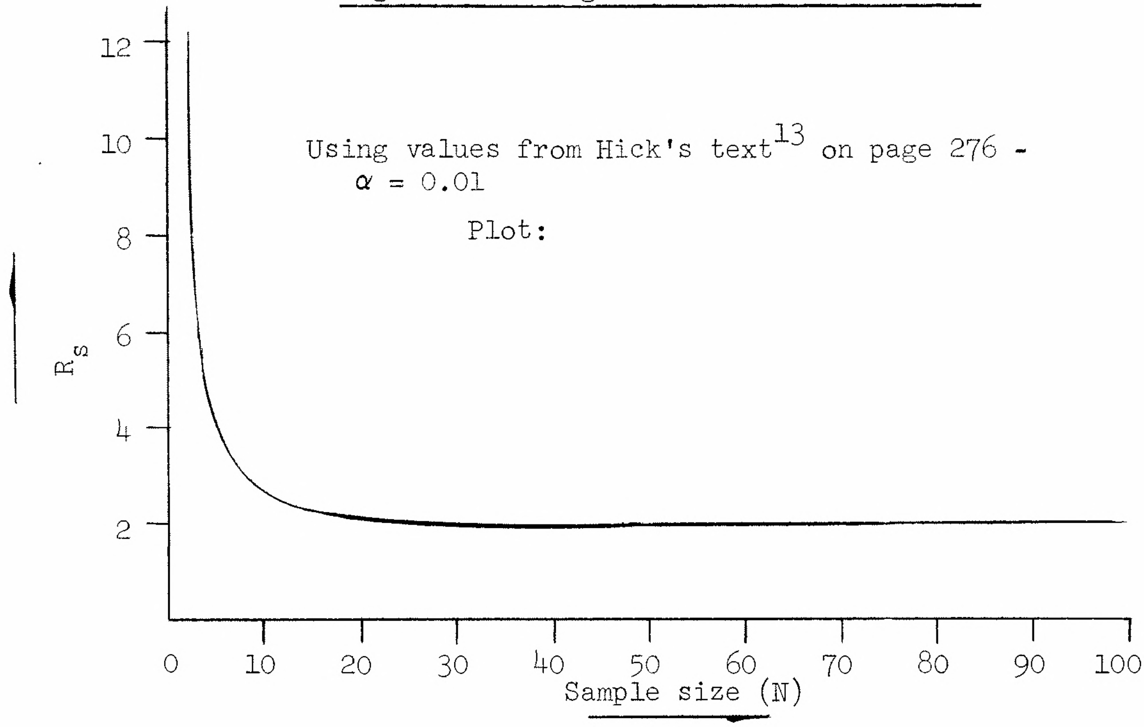
$$SS_{\text{error}} = .366058 - (.019321) - (.259382) - (.034372) - (.073935)$$

$$- (.000202) - (.016082) - (.001301) = .034000$$

$$M.S. = \frac{S.S.}{d.f.}$$

APPENDIX E

Significant Range Value for Duncan Tests



R_s = significant range

The curve resembles a hyperbola-curve of a hyperbola is:

$$R_N = \frac{\text{constant}}{N + \text{constant}} + \text{constant}$$

Using trial and error, a curve was found which describes values in the table for $N > 5$:

$$R_H = \frac{7.05}{N - 1.58} + 3.64 \qquad N > 5$$

Using this curve, a value was found for N = 500. $R_{500} = 3.654$ compared

with $N = \infty$ $\quad R_\infty = 3.64$

$\therefore$ in the tests N was assumed to be ∞

APPENDIX F

Computation of Aisle Widths Used in the Study

Angles (6) are 0°, 20°, 30°, 35°, 40°, 45°

Aisle Widths (3) will be equal to minimum width plus 6", 12", or 18"

Minimum Aisle Widths:

$$\alpha_c = \tan^{-1} \frac{(\text{pallet depth} + \text{axle to fork face})}{2(\text{pallet width} + \text{clearance})}$$

For Test Case:

$$\alpha_c = \tan^{-1}\left(\frac{54.5}{2(52)}\right) = \tan^{-1}\left(\frac{54.5}{104}\right)$$

Pallet size: 42" x 48"

Axle to fork face: 12.5" $\qquad \alpha_c = 27^\circ\ 40'$

Clearance: 4"

$$AW = T.R. + (\text{pallet depth} + \text{axle to fork face}) \cos \alpha$$
$$- 2(\text{pallet width} + \text{clearance}) \sin \alpha \text{ for } \alpha < \alpha_c$$

For 0°:

$$AW = 63 + (42 + 12.5) \cos \alpha - 2(48 + 4) \sin \alpha = 63 + 54.5 - 0$$
$$= 117.5"$$

For 20°:

$$AW = 63 + (54.5) \cos \alpha - 2(52) \sin \alpha = 63 + 51.2 - 35.6 = 78.6"$$

For 30°: $\quad > \alpha_c \quad \therefore \quad AW = 63"$

For 35°: $\quad AW = 63"$

For 40°: $\quad AW = 63"$

For 45°:* $\quad AW = 62.8"$

* see Appendix C

Aisle Widths and Aisle Facings Used in the Tests

Aisle Widths:

Angle	Min. Width	Safety Clearance 6"	12"	18"
0°	117.5"	123.5"	129.5"	135.5"
20°	78.6"	84.6"	90.6"	96.6"
30°	63.0"	69"	75"	81"
35°	63.0"	69"	75"	81"
40°	63.0"	69"	75"	81"
45°	63.0"	69"	75"	81"

Aisle Facings in Test: (42" x 48" pallet)

Angle	Aisle Facing (includes 4" clearance)
0°	52.00"
20°	55.37"
30°	60.12"
35°	63.49"
40°	67.92"
45°	73.66"

APPENDIX G

Average Times for Each Cell, Cell Totals, and Cell Sum of Square Totals

Clearance Angle	IN 6"	IN 12"	IN 18"	OUT 6"	OUT 12"	OUT 18"
0°	.2736 4.165 1.158325	.2600 3.900 ** 1.014450	.2580 3.870 * 0.999150	.2916 4.375 1.278275	.2866 4.300 * 1.234300	.2796 4.095 1.119725
20°	.2556 3.835 ** 0.983275	.2287 3.430 0.785400	.2260 3.390 0.766950	.2636 3.955 1.044875	.2390 3.585 0.858025	.2330 3.495 0.815475
30°	.2470 3.705 0.916125	.2173 3.260 0.709150	.2176 3.265 0.711150	.2636 3.955 ** 1.044075	.2256 3.385 0.765075	.2270 3.405 0.774125
35°	.2140 3.210 0.688150	.2106 3.160 0.665850	.2113 3.170 0.671300	.2253 3.380 0.762950	.2180 3.270 0.713000	.2190 3.285 0.719875
40°	.2190 3.285 0.720125	.2093 3.140 0.657550	.2023 3.035 0.614525	.2330 3.495 0.814825	.2206 3.310 0.730850	.2130 3.195 0.680975
45°	.2043 3.065 0.626875	.2013 3.020 0.608150	.2043 3.065 0.626475	.2210 3.315 0.733975	.2093 3.140 0.657650	.2173 3.260 0.709000

Times in minutes

Note: one asterisk (*) indicates a miss; a double asterisk (**), two misses.

Average Time for Each Cell Expressed in Seconds

Clearance Angle	IN 6"	IN 12"	IN 18"	OUT 6"	OUT 12"	OUT 18"
0°	16.42	15.60	15.48	17.50	17.20	16.78
20°	15.34	13.72	13.56	15.82	14.34	13.98
30°	14.82	13.04	13.06	15.82	13.54	13.62
35°	12.84	12.64	12.68	13.52	13.08	13.14
40°	13.14	12.56	12.14	13.98	13.24	12.78
45°	12.26	12.08	12.26	13.26	12.56	13.04

Model equation for ANOVA:

$$X_{ijkm} = \gamma + I_i + A_j + AW_k + IA_{ij} + IAW_{ik} + AAW_{jk} + IAAW_{ijk} + e_{m(ijk)}$$

ANOVA Table

SOURCE	d.f.	SS	MS	F value
In-out (I_i)	1	0.019321	0.019321	288.837
Angle (A_j)	5	0.259382	0.051876	774.027
Aisle width (AW_k)	2	0.034272	0.017136	256.0
I x A (IA_{ij}) interaction	5	0.001498	0.000300	4.48
I x AW (IAW_{ik}) interaction	2	0.000202	0.000101	1.51
A x AW (AAW_{jk}) interaction	10	0.016082	0.001608	24.0
I x A x AW ($IAAW_{ijk}$) interaction	10	0.001301	0.000130	1.94
Error ($E_{m(ijk)}$)	504	0.034000	0.000067	
Totals:	539	0.366058		

SOURCE: the factor we are testing for variation

d.f.: the degrees of freedom for a particular factor

SS: the sum of squares for a particular factor (Appendix D)

MS: the mean sum of squares for a particular factor (Appendix D)

F value: the value of the F statistic for the factor

Conclusions

SOURCE	Test F Value	Table F Value-conf.	Result
In-out (I_i)	288.5	$F_{1,504}$ = 6.76 99%	high sig.
Angle (A_j)	774.0	$F_{5,504}$ = 3.11 99%	very high sig.
Aisle width (AW_k)	256.0	$F_{2,504}$ = 4.71 99%	high sig.
I x A (IA_{ij}) interaction	4.47	$F_{5,504}$ = 3.11 99%	very sig.
I x AW (IAW_{ik}) interaction	1.51	$F_{2,504}$ = 1.39 75%	small sig.
A x AW (AAW_{jk}) interaction	24.0	$F_{10,504}$ = 2.32 99%	high sig.
I x A x AW ($IAAW_{ijk}$) interaction	1.94	$F_{10,504}$ = 1.83 95%	small sig.

SOURCE: the factor we are testing for variation

Test F Value: the value of the F statistic for the factor

Table F Value: the value of the F statistic found in standard tables to compare test value with to determine significance

conf.: $(1-\alpha)$ - α is probability of rejecting true hypothesis

Result: whether the test shows that a factor has significant variation among its elements or not

APPENDIX H

The Duncan Test - A Multiple Range Test

a) Varietal Means Ranked in Order:

α :	(1) 0°	(2) 20°	(3) 30°	(4) 35°	(5) 40°	(6) 45°
t_{α}:	0.2749	0.2410	0.2330	0.2164	0.2162	0.2096

b) Necessary Data from ANOUA Table:

Factor is angles, total readings/angle = 90

Error mean square (EMS_{error}) = 0.000067 d.f. = 504

c) Standard Error of a Varietal Mean:

$$s_m = \sqrt{EMS/\text{tot. readings}}$$

$$s_m = \sqrt{0.000067/90}$$

$$s_m = 0.0008628$$

d) Shortest Significant Ranges: for α = 1 percent and N = 504

p =	2	3	4	5	6
Ranges =	3.64	3.80	3.90	3.90	4.04
S.S.R. =	0.00314	0.00328	0.00336	0.00343	0.00349

e) Comparisons:

difference vs. S.S.R. (α = 0.01, N = ∞*)

6 vs. 5 = 0.0066 versus 0.00314 is a significant difference

6 vs. 4 = 0.0068 versus 0.00328 is a significant difference

6 vs. 3 = 0.0234 versus 0.00336 is a significant difference

6 vs. 2 = 0.0314 versus 0.00343 is a significant difference

*see Appendix E

6 vs. 1 = 0.0653 versus 0.00349 is a significant difference

5 vs. 4 = 0.0002 versus 0.00314 is <u>not</u> a significant difference

5 vs. 3 = 0.0168 versus 0.00328 is a significant difference

5 vs. 2 = 0.0248 versus 0.00336 is a significant difference

5 vs. 1 = 0.0587 versus 0.00343 is a significant difference

4 vs. 3 = 0.0166 versus 0.00314 is a significant difference

4 vs. 2 = 0.0246 versus 0.00328 is a significant difference

4 vs. 1 = 0.0585 versus 0.00336 is a significant difference

3 vs. 2 = 0.0080 versus 0.00314 is a significant difference

3 vs. 1 = 0.0419 versus 0.00328 is a significant difference

2 vs. 1 = 0.0339 versus 0.00314 is a significant difference

f) Results:

(1)	(2)	(3)	(4)	(5)	(6)
0°	20°	30°	$\underline{35^{\circ}}$	$\underline{40^{\circ}}$	45°

The line beneath the 35° and the 40° signifies that they are not significantly different. All of the other pairings of angles involve significant differences.

<u>Linear Regression Analysis for Times Used in the Angle-Duncan Test</u>

$$y_e = a + bx \qquad b = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{N}}{\Sigma x^2 - \frac{(\Sigma x)^2}{N}} \qquad a = \bar{y} - b\bar{x}$$

where: $\bar{t}_{\alpha} = y$

$\alpha = x$

x	y	xy	x^2
0	0.2749	0	0
20	0.2410	4.8200	400
30	0.2330	6.9900	900
35	0.2164	7.5740	1225
40	0.2162	8.6480	1600
45	0.2096	9.4320	2025
170	1.3911	37.4640	6150

$\bar{x} = 28.3333$

$\bar{y} = 0.2319$

$$b = \frac{37.4640 - \frac{(170)(1.3911)}{6}}{6150 - \frac{(170)^2}{6}}$$

$$b = \frac{37.4640 - 39.4137}{6150 - 4817}$$

$$b = \frac{-1.9497}{1333} = -.00146$$

$$a = 0.2319 - (-0.00146)(28.3333)$$

$$a = 0.2736$$

$$\therefore t_\alpha = 0.2736 - (0.00146)\alpha$$

Correlation Coefficient

$$r = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{N}}{\sqrt{\left[\Sigma x^2 - \frac{(\Sigma x)^2}{N}\right]\left[\Sigma y^2 - \frac{(\Sigma y)^2}{N}\right]}}$$

$\Sigma xy = 37.4640$

$\Sigma x = 170$

$\Sigma y = 1.3911$

$(\Sigma x)^2 = (170)^2 = 28{,}900$

$(\Sigma y)^2 = (1.3911)^2 = 1.93516$

$\Sigma x^2 = 6150$

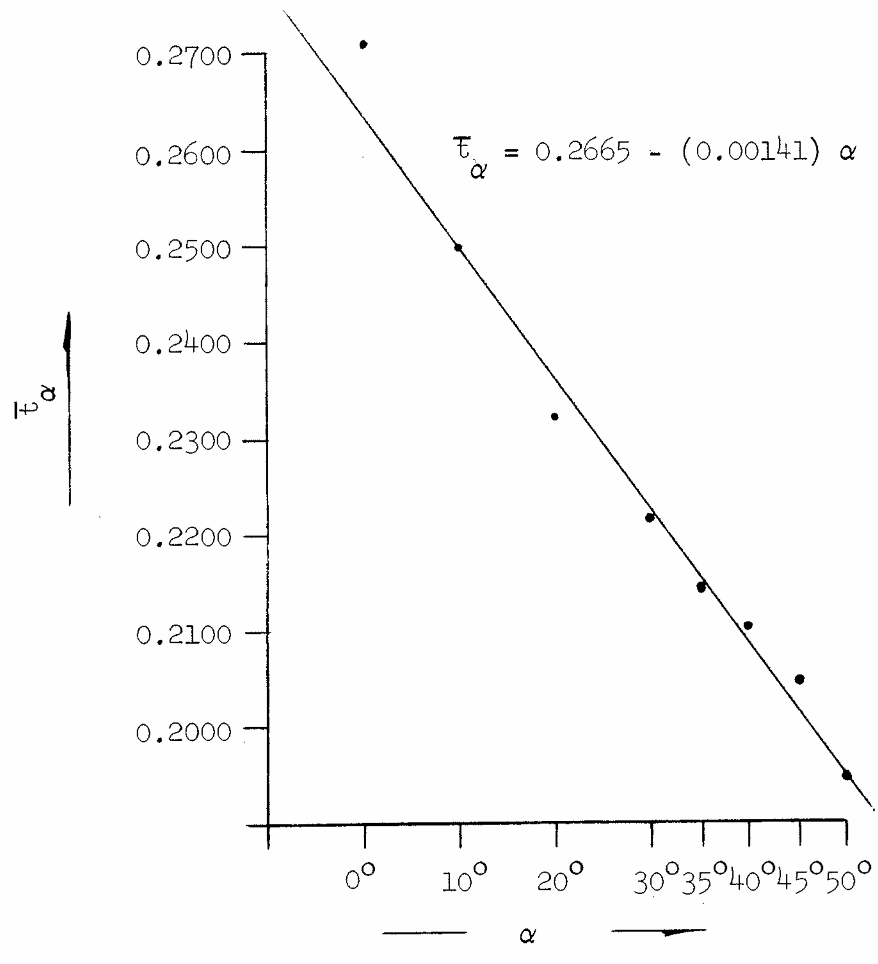
$\Sigma y^2 = 0.32544$

$$r = \frac{-1.9497}{\sqrt{[1333][.00291]}} = \frac{-1.9497}{\sqrt{3.87903}} = 1.9695$$

$$r = -.98995$$

Plot of $\overline{t}_\alpha$ Versus α for

AW = 12" and 18" Only



Linear Regression Analysis Using Only 12" AW Clear and 18" AW Clear Times

$$y_e = a + bx \qquad b = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{N}}{\Sigma x^2 - \frac{(\Sigma x)^2}{N}} \qquad a = \bar{y} - b\bar{x}$$

$$\bar{t}_\alpha = y$$

$$\alpha = x$$

$$b = \frac{36.617 - \frac{(170)(1.3589)}{6}}{6150 - \frac{(170)^2}{6}}$$

$$b = \frac{-1.833}{1333} = -0.00141$$

$$a = .2265 - (-0.00141)(28.3333)$$

$$a = 0.2665$$

$$\bar{t}_\alpha = 0.2665 - (0.00141)\ \alpha$$

x	y	xy	x^2
0	.2711	0	0
20	.2318	4.636	400
30	.2219	6.657	900
35	.2147	7.515	1225
40	.2113	8.452	1600
45	.2081	9.370	2025
170	1.3589	36.624	6150

Correlation Coefficient

$\Sigma xy = 35.624$

$\Sigma x = 170$

$\Sigma y = 1.3589$

$(\Sigma x)^2 = 28{,}900$

$(\Sigma y)^2 = 1.84661$

$\Sigma x^2 = 6150$

$\Sigma y^2 = 0.31052$

$$r = \frac{-\ 1.885}{\sqrt{[1333][.00275]}} = \frac{-\ 1.885}{1.9146}$$

$$r = -\ .98454$$

APPENDIX I

The Duncan Test - A Multiple Range Test

a) Varietal Means Ranked in Order:

AW :	(1) 6"	(2) 12"	(3) 18" (actually safety clearance)
$\bar{t}_{aw}$:	0.2427	0.2272	0.2257

b) Necessary Data from ANOUA Table:

Factor is aisle widths, total readings/AW = 180

Error mean square (EMS_{error}) = 0.000067 d.f. = 504

c) Standard Error of a Variental Mean:

$$s_m = \sqrt{EMS/\text{total readings}}$$

$$s_m = \sqrt{0.000067/180}$$

$$s_m = 0.00061$$

d) Shortest Significant Ranges: for α = 1 percent and N = 504

p =	2	3
Ranges =	3.64	3.80
S.S.R. =	0.0022204	0.0023180

e) Comparisons:

difference vs. S.S.R. (α = 0.01, N = ∞*)

1 vs. 2 = 0.0155 versus 0.00222 is a significant difference

1 vs. 3 = 0.0170 versus 0.00232 is a significant difference

2 vs. 3 = 0.0015 versus 0.00222 is not a significant difference

*see Appendix D

f) Results:

(1)	(2)	(3)
6"	12"	18"

The line beneath the 12" and 18" signifies that they are not significantly different. However, they both are significantly different from 6".

APPENDIX J

Angle	Depth of Pallets	Aisle Facing (Inches)	Add to End (Inches)	Total Bay Length (Inches)	Added Square Feet	Value of Added Space
0	10	44.0	0	1316	54.8	$ 54.80
0	8	44.0	0	1316	54.8	54.80
0	6	44.0	0	1316	54.8	54.80
0	4	44.0	0	1316	54.8	54.80
0	2	44.0	0	1316	54.8	54.80
20	10	46.8	164	1564	65.2	65.20
20	8	46.8	131	1531	63.8	63.80
20	6	46.8	98.5	1499	62.5	62.50
20	4	46.8	65.7	1466	61.0	61.00
20	2	46.8	32.8	1433	59.7	59.70
30	10	50.8	240	1770	73.8	73.80
30	8	50.8	192	1722	71.7	71.70
30	6	50.8	144	1674	69.8	69.80
30	4	50.8	96	1626	67.7	67.70
30	2	50.8	48	1578	65.7	65.70
35	10	53.9	266	1879	78.2	78.20
35	8	53.9	221	1834	76.5	76.50
35	6	53.9	165	1778	74.0	74.00
35	4	53.9	110	1723	71.8	71.80
35	2	53.9	55	1668	69.5	69.50
40	10	57.5	308	2029	84.5	84.50
40	8	57.5	247	1968	82.0	82.00
40	6	57.5	185	1906	79.4	79.40
40	4	57.5	123	1844	76.9	76.90
40	2	57.5	61.7	1783	74.3	74.30
45	10	62.3	339	2204	91.8	91.80
45	8	62.3	272	2137	89.0	89.00
45	6	62.3	204	2069	86.2	86.20
45	4	62.3	136	2001	83.4	83.40
45	2	62.3	67.8	1933	80.6	80.60

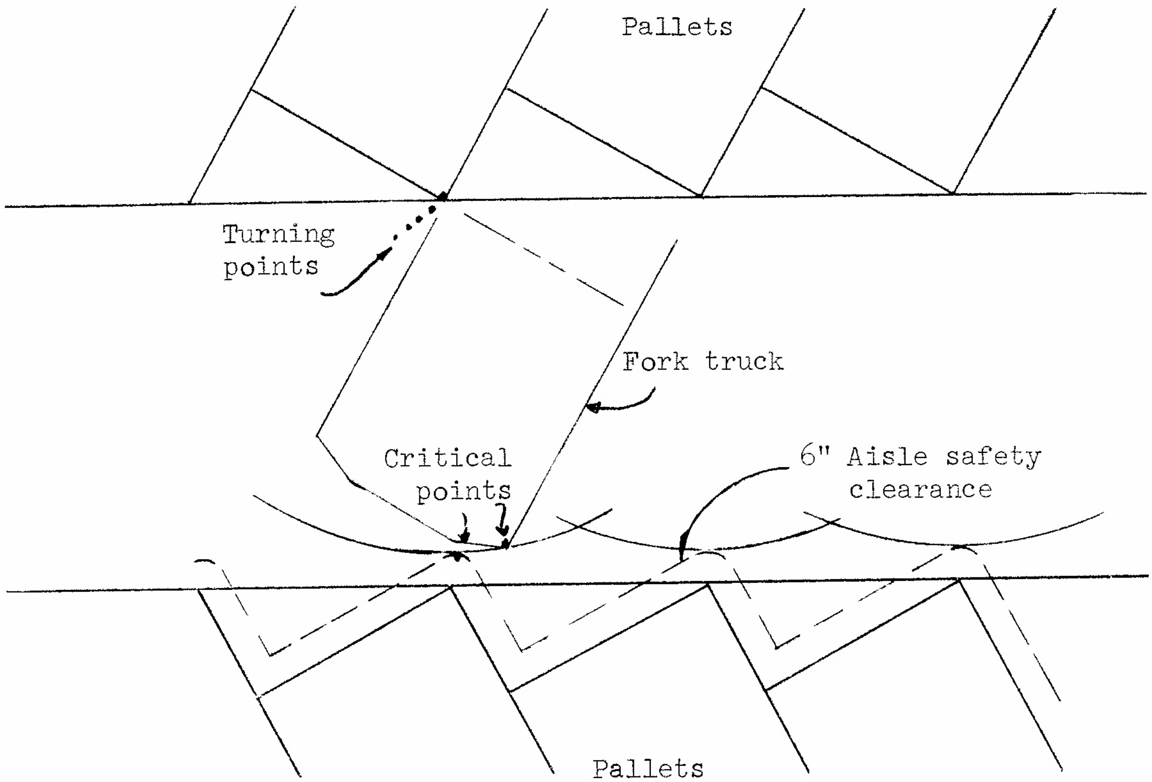
Angle	Depth of Pallets	Number of Extra End Pallets	Total Pallets in Bay	Time Saved in Minutes	Total Trips per Year to Break Even	Trips per Pallet per Year to Break Even
0	10	0	900	.0093	77300	86
0	8	0	720	.0093	77300	107
0	6	0	540	.0093	77300	143
0	4	0	360	.0093	77300	215
0	2	0	180	.0093	77300	429
20	10	78	978	.0257	33200	34
20	8	42	762	.0257	32600	43
20	6	18	558	.0257	31900	57
20	4	6	366	.0257	31100	85
20	2	0	180	.0257	30450	169
30	10	120	1020	.0338	28600	28
30	8	72	792	.0038	27800	35
30	6	36	576	.0338	27100	47
30	4	12	372	.0338	26300	71
30	2	0	180	.0338	25500	141
35	10	126	1020	.0054	190000	185
35	8	72	792	.0054	185500	234
35	6	36	576	.0054	179900	312
35	4	12	372	.0054	174200	469
35	2	0	180	.0054	169000	940
40	10	132	1032	.0111	99900	97
40	8	78	798	.0111	96800	121
40	6	36	576	.0111	93800	163
40	4	12	372	.0111	90800	244
40	2	0	180	.0111	87800	488
45	10	138	1038	.0074	162800	157
45	8	84	804	.0074	157800	196
45	6	42	582	.0074	153000	263
45	4	12	372	.0074	147800	398
45	2	0	180	.0074	144600	803

APPENDIX K

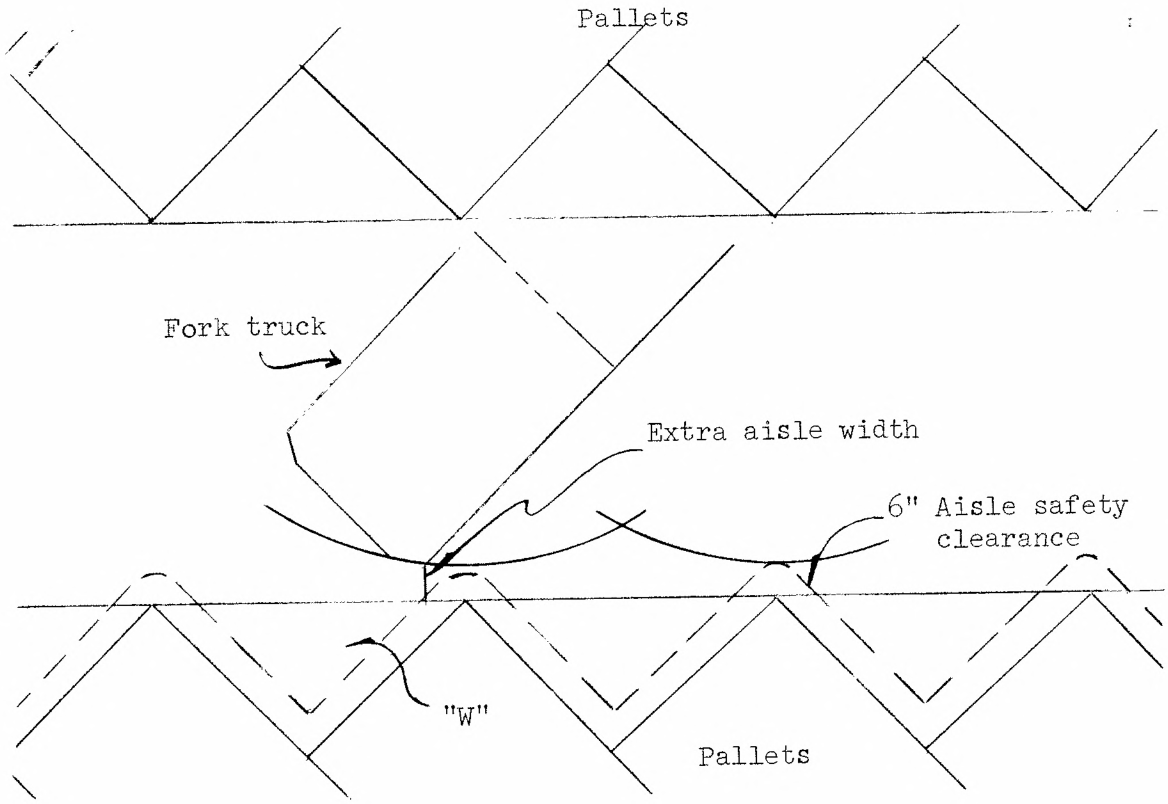
Extra Aisle Width Clearance					
Per Side of Bay (Inches)	Bay Total and Aisle Total (Inches)	Space Lost (Square Inches)	Total Space Dollar Loss per Year	Time Savings in Minutes per Trip	Total Operating Dollar Savings per Year
1.5	3.0	172.5	$ 1.20	.0107	$ .45
3.0	6.0	345.0	2.40	.0155	.65
4.5	9.0	517.5	3.59	.0165	.70
6.0	12.0	690.0	4.79	.0170	.72

This comparison was made with the example in Chapter III in mind. The aisle facing was for a 40-inch wide pallet slanted at 40 degrees. The operating cost savings were not figured at two deep, but at ten deep to check the possibility that some overall savings might be realized using deeper bays. It is evident from comparing column 4 and column 6 that no money will be saved by using wider aisles. The turnover rate used was that of the example, 18.75 trips per year.

APPENDIX L



For a "slant angle" less than α_{c2}, it is evident that the aisle width cannot be reduced without infringing on the 6-inch aisle safety clearance. (For α_{c2} see Appendix C.) α_{c2} is the angle at which the two critical points above coincide.



For a "slant angle" greater than α_{c2} it seems that a significant amount of aisle width might be saved by using the triangular wasted space, "W". However, if the position of the pallets on the near aisle is moved to the left or right a small amount, this extra space is no longer available. Mathematically, a narrower aisle is possible, but in practice it is inadvisable.

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